Optimal Insulation

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In this thesis, an optimal control optimization problem concerning the optimal placement of heat sources is solved with numerical methods. The problem is constrained by a partial differential equation and point-wise constraints and discretized with the finite element method and optimized in Matlab with a one-step factorization and an iterative interior point method. The resulting optimal heating distribution is placed as circular areas close to the windows when the penalty is reasonably high and as a thin lining along the boundary when there is no significant penalty and no windows. The number of windows drastically increases the number of necessary amount and the shape of the heating.

A similar problem is then solved as the optimization of a thin layer of insulation around a domain. This is likewise constrained by a partial differential equation and a sensitivity analysis is carried out to identify the gradient used in the interior point algorithm. The problem is discretized with finite elements and solved in Matlab. The optimal distributions show a inverse relationship between the proximity to a heat source and the level of insulation necessary. Both results show a more energy-efficient way of heating a house with a more modern approach than radiators. The focus in the thesis is on the mathematical modeling of the problem and less on the physical implementation since it is written as a bachelor project at the Institute of Mathematics at DTU.