Scaling Under Dynamic Uncertainty Using Laws of Growth

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Agenda

• Scaling and Type Range Development
• Methods for Scaling – Introduction
• Uncertainty Consideration
  • Application: Example Product
  • Dynamic Uncertainty Laws of Growth
• Conclusion
Scaling is the adaption of a product to different values of specific requirements and the transformation of data from one model’s scale to another.

[1], [8], [9]

- Scaling is used in size range development and model theory
- Size range development is according to Pahl/Beitz [1]:

By a size range we refer to technical artefacts [...] that:

- fulfil the same function
- are based on the same solution principle
- are made in varying sizes
- involve similar production processes
Scaling/Size Range Development

Examples for size ranges and scaled properties

Different kinds of scaling in size range development

following [4]
Methods for Scaling

Introduction

Dimensional Analysis

- Uses nondimensional numbers to represent similarities
- Reduces complexity through nondimensionalization
- Keeping nondimensional numbers constant leads to similar behaviour of the product

\[ Re = \frac{L \cdot v \cdot \rho}{\eta} \quad Ca = \frac{Ho}{Ne} \]

\[ Bi = \frac{h \cdot L}{\lambda} \]

Laws of Growth [1]

- Use step factors
- Express similarity laws with step factors
- Allow to design new variants of the targeted product properties quickly

\[ \phi_{i,j} = \frac{i_j}{i_0} = \frac{1 \, mm}{2 \, mm} = 0,5 \quad \phi_E = 1 \]

\[ \phi_{\text{costs}} = a_3 \phi_i^3 + a_2 \phi_i^2 + a_1 \phi_i + a_0 \]
Advantages of size range development methodology (LoG)[1]:

- Most of the design work has to be done only once per size range → cost savings.

Need for research on size range development methodology:

- No methods to calculate the impact of dynamic uncertainty with Laws of Growth.

Research Question:

How can dynamic uncertainty be taken into consideration when developing a size range using Laws of Growth?
Application

Bike disc brake (wear effects)

- Discs available with different diameters (size range)
- Semi-similarity because of interfaces (disc/hub, disc/caliper)
- $\phi_\sigma = 1$ (same utilization of the material)
- Braking reduces the cross section of the disc through abrasion $\rightarrow$ strength decreases during usage $\rightarrow$ size dependency?
- The lifetime of the disc is highly affected by uncertainty
Application

Uncertainty that has to be faced

- Geometry, tolerances
  - Caliper design, brake pads
  - Production tolerances

- Brake pressure
  - Driver’s behaviour, strength...
  - Lever

- Environmental conditions
  - humidity
  - dirt
  - temperature

- Data/information uncertainty
  - e.g. wear rate
Excursion: Uncertainty

Different types of uncertainty [16]:

- Stochastical uncertainty
- Estimated uncertainty
- Unknown uncertainty

Dynamic uncertainty depends from time or load cycles.
**Application**

*Deriving LoG for dynamic uncertainty*

\[ \phi_\sigma = \frac{\phi_F}{\phi_A} = 1 \]

\[ \phi_A = f(t, N, c_{cor}(t), F_B, A_B, \nu) \]

\[ \phi_A = f(\phi_N(t), \phi_{c_{cor}}(t), \phi_{F_B}, \phi_{A_B}, \phi_\nu) \]

\[ 1 = \frac{\phi_F}{\phi_A} \]

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$t =$ time  

$N =$ revolutions (wheel, braking)  

$c_{corr} =$ corrosion rate  

$F_B =$ brake force on capiler  

$A_B =$ area of brake pad  

$\nu =$ rel. speed of brake pad to disc
Application

Computing the product's behaviour

Dynamic deviation of input parameters

Margin of deviation known (Estimated Uncertainty)

Scenario based dynamic Laws of Growth

Approximation of dynamic extremal values

Probability distribution known (Stochastical Uncertainty)

Monte Carlo Simulation with dynamic dependencies

Dynamic probability distribution of target parameter

\[ [i_{\text{min}}(t), i_{\text{max}}(t)] \]

\[ \phi_o(t) \]

\[ \phi_i(t) \]

\[ P \rightarrow t \]

\[ \phi_i \]

\[ P \rightarrow t \]

\[ \phi_o \]

i=input parameter, o=output parameter
Conclusion

Dynamic uncertainty quantification with LoG

- Quantification of size and time dependent uncertainty in size range development is possible
- Leads to better knowledge about product
- Less safety margin needed/higher level of safety achieved
- Categorisation of methods done

\[
P_{\phi_i} = f \left( P_{\phi_j}, t \right)
\]

Information needed

Ability to handle dynamic uncertainty
Thank you for your attention!

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References (I)

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