A Solution for a Tidal Leaky Aquifer Extending Finite Distance under the Sea

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ABSTRACT

An analytical solution is derived to investigate the influences of leakage and roof length on the groundwater head of leaky aquifer extending finite distance under the sea. The leakages from the offshore and inland aquitards are two dominant factors controlling the fluctuation of groundwater level. The influence distance from the coast decreases significantly with the leakage of the inland aquitard. The fluctuation of groundwater level in the inland part of the leaky aquifer increases significantly with the leakage of the offshore aquitard. The dynamic effect of the water-table fluctuation on the unconfined aquifer increases with the roof length and then approaches a constant value when the roof length is greater than a threshold value.

1. INTRODUCTION

In most coastal areas, groundwater and seawater are hydraulic connected. Since the 1950s, research on the dynamic interaction between groundwater and seawater has attracted much attention because of various coastal hydrological, engineer, and environmental problems. These include coastal aquifer parameter estimation, beach dewatering, marine retaining structures, and seawater intrusion. An analytical solution is derived to investigate the influences of leakage and roof length on the groundwater head of leaky aquifer extending finite distance under the sea. The present solution can reduce to the Li and Jiao's solution (2001) when without considering the water-table fluctuation in the unconfined aquifer. This paper will focus on groundwater dynamics in response to the tidal fluctuation in a coastal aquifer system. The leakage of the offshore aquitard, which may be formed by sedimentary depositional process of the offshore current, is assumed different from that of the inland aquitard. Thus, the leakage effects of both inland and offshore aquitards on the head distribution of the tidal leaky confined aquifer are considered and discussed. This solution differs form that of Li and Jiao (2001) in two situations: (1) the offshore and inland parts of this layer have different hydraulic properties and (2) the water table in the unconfined aquifer fluctuates with tide. An attempt is made to understand the influence of those two situations on the behavior of the groundwater level fluctuations in the inland part of the leaky aquifer. Based on this solution, the joint effects of various parameters, such as dynamic effect of water table fluctuation and the leakages of the inland and offshore, on the behavior of the groundwater level fluctuations in the inland part of the leaky confined aquifer can be clearly explored.

2. PROBLEM SETUP AND BOUNDARY CONDITIONS

Consider a coastal aquifer system with an unconfined aquifer, a leaky aquifer, and an aquitard between them as shown in Figure 1. The effect of tidal fluctuations on both the unconfined and the leaky aquifer are considered. These two aquifers interact with each other through leakage. The unconfined aquifer terminates at the coast while the aquitard and the leaky aquifer extend finite distance (l) under the sea. The bottom of the leaky aquifer is impermeable. The leakages of the offshore and inland aquitards are independent parameters.

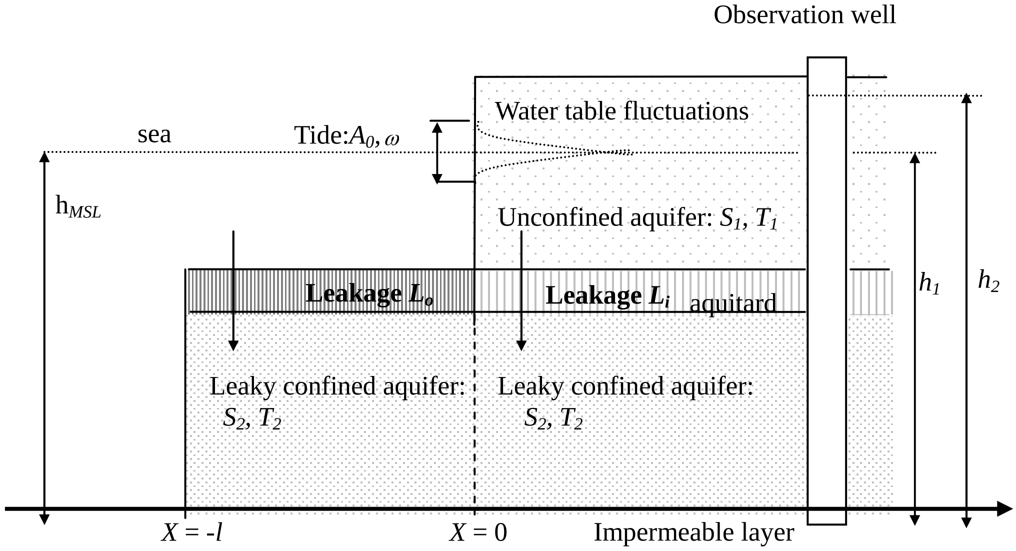


Figure 1. Schematic diagram of a leaky aquifer.

The origin of the x-axis is at the intersection of the mean sea surface and the beach face. The x-axis is horizontal, positive landward, and perpendicular to the coastal line. Assume that the aquifer material is homogeneous and isotropic and the thickness of the unconfined aquifer is very large when compared to the magnitude of the tidal fluctuations, therefore allowing linearity of the governing flow equations. The flow velocity in the leaky aquifer is essentially horizontal, and there is a vertical leakage through the aquitard. The initial hydraulic head in the whole system is uniform and equals h_{MSL}, which is the distance from the groundwater level to any convenient reference. In addition, the aquitard storage is assumed negligible and leakage is linearly proportional to the difference in head between the two aquifers, the unconfined aquifer and leaky confined aquifer (Bear 1987; Li 2001). Under these assumptions the governing equations of the head fluctuation for the inland unconfined and the leaky confined aquifer ($x > 0$) are respectively (Bear 1987; Li 2001)

$$S_1 \frac{\partial h_1}{\partial t} = T_1 \frac{\partial^2 h_1}{\partial x^2} + L_i (h_2 - h_1) \qquad (1)$$

$$S_2 \frac{\partial h_2}{\partial t} = T_2 \frac{\partial^2 h_2}{\partial x^2} + L_i (h_1 - h_2) \qquad (2)$$

and for offshore aquifer ($x < 0$) is

$$S_2 \frac{\partial h_2}{\partial t} = T_2 \frac{\partial^2 h_2}{\partial x^2} + S_2 T_e \frac{d h_s}{dt} + L_o (h_s - h_2) \tag{3}$$

where h_1 and h_2 are the hydraulic heads in the unconfined and the leaky aquifers, respectively; h_s is hydraulic head of the sea tide; T_e is tidal efficiency, which reflects the fluctuation of groundwater level caused by compression of both the aquifer skeleton and groundwater due to the tidal loading above the offshore aquitard (Li 2001); S_1 and S_2 , T_1 and T_2 are the storativities and transmissivities of these two aquifers, respectively; and the leakages, L_o and L_i, are defined as the ratio of the hydraulic conductivity of the aquitard to the thickness of the aquitard. Note that the hydraulic conductivity and/or thickness of the inland aquitard may differ from those of the offshore aquitard due to different depositional sediment facies (Reineck 1980). L_o and L_i are the leakages of the offshore and inland aquitards, respectively.

The tidal boundary may be written as

$$h_1(0,t) = h_s(t) = h_{MSL} + A_0 \cos(\omega \cdot t) \tag{4}$$

$$h_2(-l,t) = h_s(t) = h_{MSL} + A_0 \cos(\omega \cdot t) \tag{5}$$

The continuity conditions of the hydraulic head and flux at x = 0 respectively require

$$\lim_{x \downarrow 0} h_2(x,t) = \lim_{x \uparrow 0} h_2(x,t) \tag{6}$$

$$\lim_{x \downarrow 0} \frac{\partial h_2(x,t)}{\partial x} = \lim_{x \uparrow 0} \frac{\partial h_2(x,t)}{\partial x} \tag{7}$$

where $h_1(0,t)$ is the hydraulic head at $x = 0$, $-l$ is the distance extending under the sea, A_0 is the amplitude of the tidal change, and ω is the tidal speed. Also $\omega = 2\pi/t_0$ where t_0 is the tidal period. The boundary conditions for (1) and (2) on the inland side may be expressed as

$$\lim_{x \to \infty} \frac{\partial h_1}{\partial x} = 0 \tag{8}$$

$$\lim_{x \to \infty} \frac{\partial h_2}{\partial x} = 0 \tag{9}$$

which state that the slopes of the hydraulic head approach zero at the remote boundary.

3. ANALYTICAL SOLUTION

Let $H_1(x, t)$ and $H_2(x, t)$ be complex functions of the real variables x and t that satisfy (1) ~ (9). Assume that $h_1(x, t)$ and $h_2(x, t)$ are the solutions to (1) ~ (9) and follow that

$$h_1(x, t) = h_{MSL} + Re\,[H_1(x, t)] \tag{10}$$

$$h_2(x, t) = h_{MSL} + Re\,[H_2(x, t)] \tag{11}$$

where Re denotes the real part of the complex expression and $i = \sqrt{-1}$.

Now suppose

$$H_1(x,t) = A_0 X_1(x) e^{-i\omega t} \tag{12}$$

$$H_2(x,t) = A_0 X_2(x) e^{-i\omega t} \tag{13}$$

where $X_1(x)$ and $X_2(x)$ are unknown functions of x. Substituting (12) and (13) into those nine equations, which $H_1(x, t)$ and $H_2(x, t)$ satisfy, and dividing the results by $A_0 e^{-i\omega t}$, yield the result for the inland aquifer ($x > 0$) as

$$X''_1(x) + \frac{i\omega S_1 - L_i}{T_1} X_1(x) + \frac{L_i}{T_1} X_2(x) = 0 \quad (14)$$

$$X''_2(x) + \frac{i\omega S_2 - L_i}{T_2} X_2(x) + \frac{L_i}{T_2} X_1(x) = 0 \quad (15)$$

and the results for offshore aquifer ($x < 0$) as

$$X''_2(x) + \frac{i\omega S_2 - L_o}{T_2} X_2(x) = \frac{i\omega T_e S_2 - L_o}{T_2} \quad (16)$$

Furthermore, the tidal boundaries of (4) and (5) may be rewritten as

$$X_1(0) = 1 \quad (17)$$

$$X_2(-l) = 1 \quad (18)$$

The continuity conditions of (6) and (7) may be replaced as

$$\lim_{x \downarrow 0} X_2(x) = \lim_{x \uparrow 0} X_2(x) \quad (19)$$

$$\lim_{x \downarrow 0} X'_2(x) = \lim_{x \uparrow 0} X'_2(x) \quad (20)$$

The boundary conditions of (8) - (9) may also be rewritten as

$$X'_1(+\infty) = 0 \quad (21)$$

$$X'_2(+\infty) = 0 \quad (22)$$

Thus, the general solutions to (14) ~ (16) for inland aquifer ($x > 0$) are

$$X_1(x) = \alpha_1 e^{-\lambda_1 x} + \alpha_2 e^{-\lambda_2 x} \quad (23)$$

$$X_2(x) = \alpha_1 \beta_1 e^{-\lambda_1 x} + \alpha_2 \beta_2 e^{-\lambda_2 x} \quad (24)$$

and for offshore aquifer ($x < 0$) is

$$X_2(x) = \alpha_3 e^{\lambda_3 x} + \alpha_4 e^{-\lambda_3 x} + \beta_3 \quad (25)$$

where variables α_1, α_2, α_3, α_4, β_1, β_2, β_3, λ_1, λ_2 and λ_3 are defined as

$$\alpha_1 = \frac{D_1}{D} \quad (26a)$$

$$\alpha_2 = \frac{D_2}{D} \quad (26b)$$

$$\alpha_3 = \frac{D_3}{D} \quad (26c)$$

$$\alpha_4 = \frac{D_4}{D} \quad (26d)$$

$$\begin{aligned} D = {} & e^{-\lambda_3 l} (\beta_1 \lambda_1 - \beta_2 \lambda_2 - \beta_1 \lambda_3 + \beta_2 \lambda_3) \\ & + e^{\lambda_3 l} (-\beta_1 \lambda_1 + \beta_2 \lambda_2 - \beta_1 \lambda_3 + \beta_2 \lambda_3) \end{aligned} \quad (26e)$$

$$\begin{aligned} D_1 = {} & e^{-\lambda_3 l} (-\beta_2 \lambda_2 + \beta_2 \lambda_3 - \beta_3 \lambda_3) \\ & + e^{\lambda_3 l} (\beta_2 \lambda_2 + \beta_2 \lambda_3 - \beta_3 \lambda_3) \\ & + 2 \beta_3 \lambda_3 - 2 \lambda_3 \end{aligned} \quad (26f)$$

$$\begin{aligned} D_2 = {} & e^{-\lambda_3 l} (\beta_1 \lambda_1 - \beta_1 \lambda_3 + \beta_3 \lambda_3) \\ & + e^{\lambda_3 l} (-\beta_1 \lambda_1 - \beta_1 \lambda_3 + \beta_3 \lambda_3) \\ & - 2 \beta_3 \lambda_3 + 2 \lambda_3 \end{aligned} \quad (26g)$$

$$\begin{aligned} D_3 = {} & \beta_1 \lambda_1 - \beta_2 \lambda_2 - \beta_1 \lambda_3 + \beta_2 \lambda_3 \\ & - \beta_1 \beta_3 \lambda_1 + \beta_2 \beta_3 \lambda_2 + \beta_1 \beta_3 \lambda_3 - \beta_2 \beta_3 \lambda_3 \\ & + e^{\lambda_3 l} (-\beta_1 \beta_2 \lambda_1 + \beta_1 \beta_2 \lambda_2 + \beta_1 \beta_3 \lambda_1 - \beta_2 \beta_3 \lambda_2) \end{aligned} \quad (26h)$$

$$D_4 = -\beta_1\lambda_1 + \beta_2\lambda_2 - \beta_1\lambda_3 + \beta_2\lambda_3 + \beta_1\beta_3\lambda_1 - \beta_2\beta_3\lambda_2 + \beta_1\beta_3\lambda_3 - \beta_2\beta_3\lambda_3 + e^{-\lambda_3 l}(\beta_1\beta_2\lambda_1 - \beta_1\beta_3\lambda_1 - \beta_1\beta_2\lambda_2 + \beta_2\beta_3\lambda_2) \tag{26i}$$

$$\beta_1 = \frac{L_i - T_1B_1 - i\omega S_1}{L_i} \tag{26j}$$

$$\beta_2 = \frac{L_i - T_1B_2 - i\omega S_1}{L_i} \tag{26k}$$

$$\beta_3 = \frac{L_o - i\omega S_2 T_e}{L_o - i\omega S_2} \tag{26l}$$

$$\lambda_1 = \sqrt{B_1} \tag{26m}$$

$$\lambda_2 = \sqrt{B_2} \tag{26n}$$

$$\lambda_3 = \left(\frac{L_o - i\omega S_2}{T_2}\right)^{0.5} \tag{26o}$$

variables B_1 and B_2 are respectively defined as

$$B_1 = -a - \sqrt{a^2 - b} \tag{26p}$$

$$B_2 = -a + \sqrt{a^2 - b} \tag{26q}$$

and variables a and b are respectively defined as

$$a = \frac{L_i}{2T_1T_2}\left(\frac{i\omega T_1S_2}{L_i} + \frac{i\omega T_2S_1}{L_i} - T_1 - T_2\right) \tag{26r}$$

$$b = -\frac{L_i}{T_1T_2}\left(\frac{\omega^2 S_1S_2}{L_i} + i\omega S_1 + i\omega S_2\right) \tag{26s}$$

Similarly, the solutions of $h_1(x, t)$ and $h_2(x, t)$ for inland aquifer ($x > 0$) are

$$h_1(x,t) = h_{MSL} + \mathrm{Re}[A_0(\alpha_1 e^{-\lambda_1 x} + \alpha_2 e^{-\lambda_2 x})\, e^{-i\omega t}] \tag{27}$$

$$h_2(x,t) = h_{MSL} + \mathrm{Re}[A_0(\alpha_1\beta_1 e^{-\lambda_1 x} + \alpha_2\beta_2 e^{-\lambda_2 x})\, e^{-i\omega t}] \tag{28}$$

and that for offshore aquifer ($x < 0$) is

$$h_2(x,t) = h_{MSL} + \mathrm{Re}[A_0(\alpha_3 e^{\lambda_3 x} + \alpha_4 e^{-\lambda_3 x} + \beta_3)\, e^{-i\omega t}] \tag{29}$$

4. RESULTS AND DISCUSSION

Equations (27) and (28) are respectively the solution for the groundwater heads in inland unconfined and the leaky confined aquifer and (29) is the solution for offshore confined aquifer. Since most field studies on coastal aquifers focus on the inland part of the aquifer and there may be no observation of groundwater heads available in the offshore area. Thus, only the groundwater heads in inland part of the aquifer are discussed in this paper.

4.1. Joint effects of water table fluctuation and roof length. The first case is used to demonstrate how water table fluctuation and roof length (l) under the sea influence the tidal fluctuations. Case 1 uses the following parameters: T_1 = 2000 m^2/d, T_2 = 2000 m^2/d, S_1 = 0.3, S_2 = 0.0001, T_e = 0.0, A = 0.65 m, $L_i = L_o$ and w = 0.253 /d. In the case 1, the solid lines of Fig. 2 clearly show that the normalized groundwater amplitude at x = 0, HA_{x0}, in the leaky aquifer decreases as the roof length increases and their decrease rates increases with the leakage. In Fig. 2, the dash lines represent the solution when $S_1 \to \infty$ or $T_1 \to 0$ and the symbol □ stands for Li and Jiao's solution (2001). Figure 2 displays that this newly derived solution when without considering the fluctuation of groundwater level in the unconfined aquifer reduces to Li and Jiao's solution (2001). Figure 2 also indicate that the dynamic effect of the water table fluctuation in the unconfined aquifer increases with l and approaches a constant value when l is greater than a threshold value. In addition, the dynamic effect increases with the leakage ($L_i = L_o$).

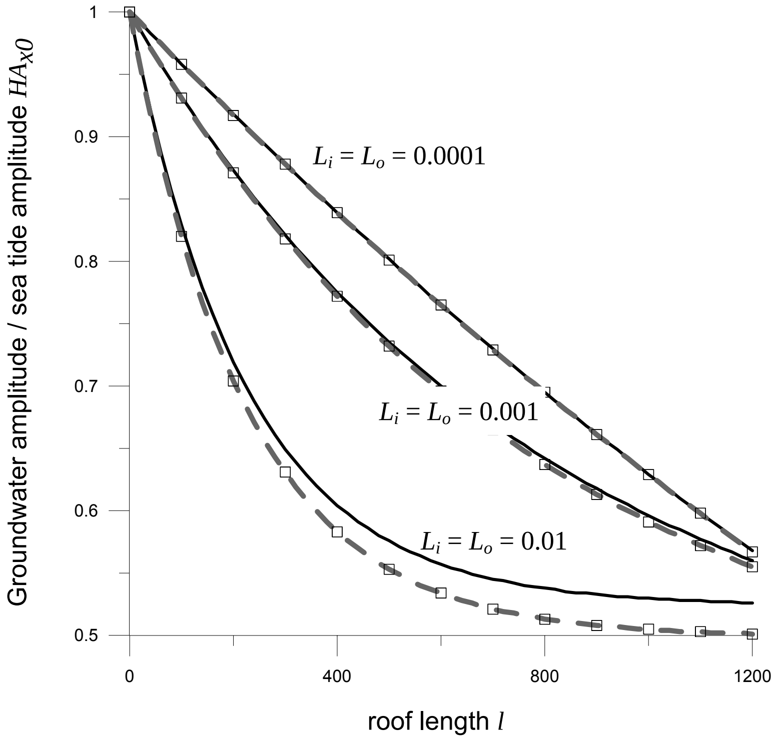
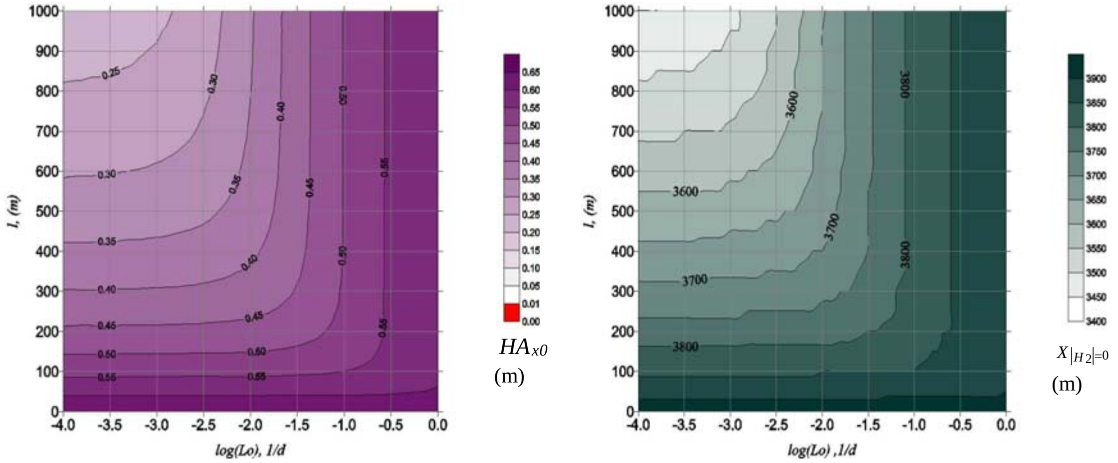


Figure 2. The curves of HA_{x0} versus l when the leakage ($L_i = L_o$) varies from 0.0001 to 0.01 /d. The solid lines denote the present solution, the dash lines represent the present solution when $S_1 \to \infty$ or $T_1 \to 0$, and the symbol □ stands for Li and Jiao's solution (2001) of without considering the fluctuation of groundwater level in the unconfined aquifer.

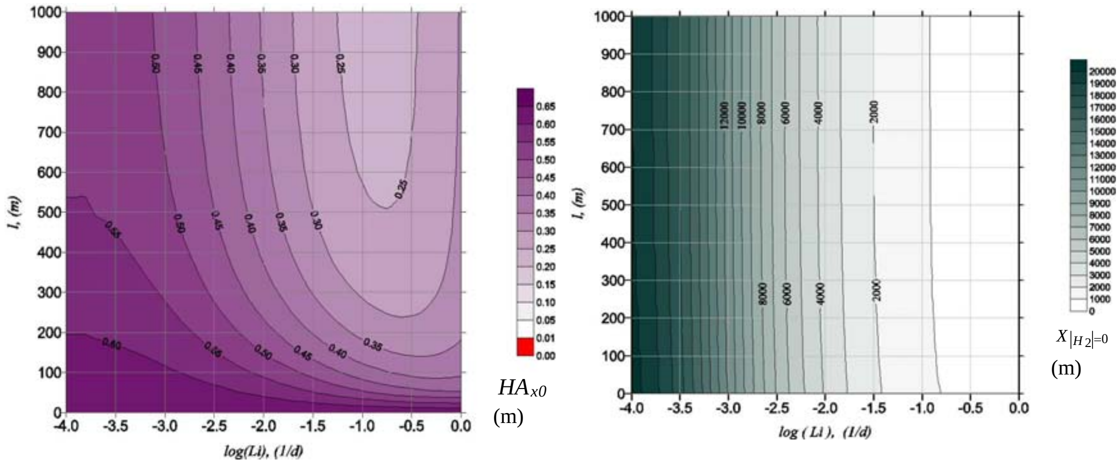
4.2. Effects of leakage (L_o). The second case is used to demonstrate how leakage (L_o) influences the tidal fluctuations. Case 2 uses the following parameters: $T_1 = 2000$ m^2/d, $T_2 = 2000$ m^2/d, $S_1 = 0.3$, $S_2 = 0.0001$, $T_e = 0.0$, $L_i = 0.01$ /d, $A = 0.65$ m and $w = 0.253$ /d. In the case 2, Fig. 3(a) displays how L_o influences the threshold value. The fluctuation in the inland part of the leaky aquifer increases significantly with the leakage (L_o) of the offshore aquitard. When $L_o = 0.1$ /d, the roof length is greater than a threshold value ($l > 400$ m), the maximum groundwater amplitude (HA_{x0}) in the leaky aquifer approaches 0.5 m and is no longer sensitive to l. Similarly, when $l = 100$ m, the L_o is smaller than a threshold value ($L_o < 0.01$ /d), the maximum groundwater amplitude (HA_{x0}) in the leaky aquifer approaches 0.54 m and is no longer sensitive to L_o. Fig. 3(a) indicates that the threshold values decrease with the leakage (L_o). Fig. 3(b) shows that the L_o has similar influence to the effect distance ($x_{|H2|=0}$) of the fluctuation in inland part.



(a) The maximum groundwater amplitude (HA_{x0}) contour in inland part

(b) The effect distance contour of the groundwater head fluctuation from the coastline in inland part

Figure 3. The maximum groundwater amplitude (HA_{x0}) and the effect distance ($X_{|H2|=0}$) contours with roof length (*l*) and leakage (L_o).



(a) The maximum groundwater amplitude (HA_{x0}) contour in inland part

(b) The effect distance contour of the groundwater head fluctuation from the coastline in inland part

Figure 4. The maximum groundwater amplitude (HA_{x0}) and the effect distance ($X_{|H2|=0}$) contours with roof length (*l*) and leakage (L_i).

4.3. Effects of leakage (L_i). The third case is used to demonstrate how leakage (L_i) influences the tidal fluctuations. Case 3 uses the following parameters: T_1 = 2000 m^2/d, T_2 = 2000 m^2/d, S_1 = 0.3, S_2 = 0.0001, T_e = 0.0, L_o = 0.01 /d, A = 0.65 m, and w = 0.2618 /hr. In the case 3, Fig. 4(a) shows that the maximum groundwater amplitude (HA_{x0}) in the leaky aquifer decreases with the leakage (L_i), and then increases with the leakage (L_i) when $l > 0$. The maximum groundwater amplitude (HA_{x0}) in the leaky aquifer is more sensitive to the change of l when L_i ranging between 0.1 and 0.01 /d than those of L_i ranging between 0.01 and 0.0001 /d. The effect distance ($X_{|H_2|=0}$) is more sensitive to the change of L_i than that of L_o obtaining when comparing Fig. 3(b) with 4(b). Fig. 4(b) also shows that the effect distance ($X_{|H_2|=0}$) rapidly decreases as L_i increase for any roof length (l).

5. CONCLUSIONS

An analytical solution is derived to investigate the influences of leakage and roof length on tidal responses in a coupled coastal aquifer system consisting of an unconfined aquifer, aquitard, and leaky aquifer. The unconfined aquifer terminates at the coast while the aquitard and leaky aquifer extend finite distance under the sea. The present solution can reduce to the Li and Jiao’s solution (2001) when without considering the water-table fluctuation in the unconfined aquifer. Detailed investigations are carried out to explore the influence of various parameters, such as dynamic effect of water table fluctuation and the leakages of the inland and offshore, on the behavior of the groundwater level fluctuations of the inland leaky aquifer. Both offshore and inland leakages are two dominant factors controlling the groundwater fluctuations. The influence distance from the coast decreases significantly with the leakage (L_i) of the inland aquitard. The groundwater level fluctuation in the inland part of the leaky aquifer increases significantly with the leakage (L_o) of the offshore aquitard. The dynamic effect of the water-table fluctuation on the unconfined aquifer increases with the roof length and then approaches a constant value when the roof length is greater than a threshold value.

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