COMPARISON OF A MICRO-SCALE AND A MESO-SCALE MODEL CONCEPT FOR TWO-PHASE FLOW IN FRACTURED-POROUS MEDIA

SONG PHAM VAN¹, LEOPOLD STADLER¹, REINHARD HINKELMANN¹

¹ Department of Water Resources Management and Hydroinformatics, Institute of Civil Engineering, Technische Universität Berlin, Germany

Abstract

Two-phase flow processes in porous media are of considerable importance in hydro- and environmental sciences. In this contribution, we discuss and compare two different model concepts for two-phase flow in fractured-porous media, which are implemented in the numerical simulator MUFTE-UG. The two different model concepts are the so-called micro-scale and the meso-scale model concept. In the micro-scale model concept, the fractures are discretized as two-dimensional elements in the two-dimensional domain. The mesh is highly resolved in the fracture as well as in its close surroundings. In the meso-scale model concept, fractures are modeled as one-dimensional elements in a two-dimensional domain. They are considered as elements of lower dimension. The mathematical model for two-phase flow in fractured porous media including the governing equations as well as the numerical algorithm is briefly explained. The two model concepts have been compared in a case study dealing with water infiltration into a domain with a vertical fracture. Overall, they agree well. Differences are caused by different mesh resolutions of the fracture-matrix surroundings. Future work includes the simulation of infiltration experiments carried out in a laboratory and the transition to the field scale.

1. INTRODUCTION

Two-phase flow processes in porous media occur in a wide variety of hydro- and environmental engineering fields such as water-gas flow in coal mining areas, salt-water intrusion problems in coastal areas, seepage processes through dikes or water-infiltration processes in hillslopes. The description of two-phase flow in fractured porous media and macropores is a very challenging problem because of the heterogeneous structures, the very different process speeds and the multiples scales involved which all together lead to highly non-linear equations (see Hinkelmann (2005), Helmig (1997), Simunek (2003), Reichenberger (2004)).

2. MODEL CONCEPTS FOR FLOW IN FRACTURED-POROUS MEDIA

2.1. General information. Fractured porous media are very complex systems which consist of different parts like fault zones, fractures, fissures and the surrounding rock matrix (see Hinkelmann (2005), Simunek (2003)). In this work, we discuss and compare two different model concepts for two-phase flow in fractured-porous media, a micro-scale
model concept and meso-scale model concept. With fractured-porous media, model concepts for the combined consideration of processes in the fracture network and the matrix can generally be distinguished into discrete and equivalence model concept (see Helmig (1997), Hinkelmann (2005), Bogdanov (2003)).

In discrete model concepts, the fractures are treated as a porous medium, generally with a much higher permeability and lower storage capacity compared to the matrix. When the combined model approach is applied, fractures as well as the matrix are taken into account. The fractures are discretized as elements of lower dimension, for example, a one-dimensional fracture in a two-dimensional domain. The processes are dominated by the fractures in a way that makes homogenization impossible.

In equivalence model concepts (or continuum model concepts), the domain is homogenized based on assumptions about the regularity of fractures. It is assumed that an REV cannot only be obtained for the porous medium but also for the fractured system. In these approaches, single-continuum, double-continuum and multi-continuum approaches are distinguished. The continua are coupled by exchange parameters (see Figure 1).

Additionally, the fractures or macropores can be excluded from the porous medium. The flow in these areas can be modeled as pipe flow, which is coupled to the porous media flow along the common boundaries. The exchange (leakage) parameters between 'pipe' and matrix must be determined.

**Figure 1.** Model concepts for fractured porous media (after Hinkelmann (2005))
2.2. Micro-scale model concept. The fractured porous media is defined in two different properties, the fractures and the matrix. It is characterized by a high permeability within the fractures and a low permeability within the matrix. The so-called micro-scale model concept takes the fracture and the matrix into account as elements of the same dimension. Here, the fractures are set as the two-dimensional elements in the two-dimensional domain. Therefore, the mesh is highly resolved in the fracture as well as in its close surroundings (see Figure 2).

![Figure 2. Mesh for the micro-scale model concept (left), zoom around a fracture (right)](image)

2.3. Meso-scale model concept. On meso and large scales, the micro-scale model concept cannot be used in a larger domain due to the limitations of the CPU time. The meso-scale model concept is based on the combined model approach, i.e. fractures are considered as elements of lower dimension, for example as one-dimensional elements in a two-dimensional domain. Generally, the mesh resolution of the fracture-matrix space is coarser compared to the micro-scale model concept (see Figure 3). However, to compare both model concepts, approximately the same mesh resolution is chosen here.

![Figure 3. Mesh for the meso-scale model concept, (left), zoom around a fracture (right)](image)
In the following we will check whether we find back the results of the micro-scale model concept in the meso-scale model concept.

3. MATHEMATICAL MODELING FOR TWO-PHASE FLOW IN FRACUTRED POROUS MEDIA

The model concept of two-phase flow in porous media assumes that the two fluids, here water (w) and air (a), are not miscible in each other and that mass transfer processes, for example evaporation or condensation, are negligible. The processes are averaged in the REV (see Figure 4). The following equations and laws hold for the matrix as well as for the fractures.

\[
\frac{\partial}{\partial t} (S_\alpha \phi \rho_\alpha) + \text{div} (\rho_\alpha \mathbf{v}_\alpha) - \rho_\alpha q_\alpha = 0
\]  

(1)

The validity of the extended Darcy law is assumed:

\[
\mathbf{v}_\alpha = -\frac{k_{r\alpha}}{\mu_\alpha} K (\text{grad} p_\alpha - \rho_\alpha \mathbf{g})
\]  

(2)

In these equations, \( \phi \) stands for the porosity, \( S \) for the saturation, \( \rho \) for the density, \( t \) for the time, \( q \) for a sink / source term, \( \mathbf{v} \) for the Darcy-velocity vector, \( k_r \) for the relative permeability, \( \mu \) for the dynamic viscosity, \( K \) for the permeability tensor, \( p \) for the pressure and \( \mathbf{g} \) for the gravity vector.

Two further conditions are required to close the system of equations. The saturations, which describe the ratio of the fluid volume to the volume of the void space, add up to one:

\[
S_w + S_a = 1
\]  

(3)

At the interface between the two fluids, a jump in the pressure occurs which is given by the capillary pressure \( p_c \):

\[
p_a - p_w = p_c
\]  

(4)

The capillary pressure and the relative permeability are (highly) non-linear functions of the saturations and must be determined experimentally; they are called constitutive relationships.

![Figure 4. REV concept.](image-url)
The two-phase flow model concepts are implemented in the modeling system MUFTE-UG which is a combination of MUFTE and UG. MUFTE stands for MUltiphase Flow, Transport and Energy model. UG is the abbreviation for Unstructured Grids. The MUFTE parts mainly contains the physical model concepts and discretization methods for isothermal and non-isothermal multiphase / multicomponent flow and transport processes in porous and fractured-porous media, while UG provides the data structures and fast solvers based on parallel, adaptive Multigrid Methods (see e.g. Hinkelmann (2005), Reichenberger (2004), Pham-Van (2004)).

4. SIMULATION RESULTS

The domain shown in Figures 2 and 3 is chosen for the simulations. Water infiltration processes are investigated in the system with the Dirichlet boundary condition (BC) on the top. The water height is set to 1cm equivalent to pressure of 100100Pa. The other sides of the domain are closed (Neumann BC) (see Figure 5). The domain has the permeability $K_m = 10^{-12}m^2$ and the porosity $\phi = 0.57$, and the fracture has the permeability $K_{fr} = 10^{-9}m^2$, the porosity $\phi = 0.9$ and the fracture width $b = 1cm$. The fluid properties of water are $\rho_w = 10^3kg/m^3$, $\mu_w = 10^3Pa\cdot s$ and of air are $\rho_a = 1.2kg/m^3$, $\mu_a = 1.02x10^{-5}Pa\cdot s$ The constitutive relationships of Brooks Corey are chosen with the parameters $\lambda = 2.0$, $p_d = 1000Pa$. Initially, the air saturation is $S_{a0} = 0.9$. Additionally, residual water and air (or gas) saturations must be given $S_{wr} = 0.1$, $S_{ar} = 0.01$.

![Figure 5. Model setup](image)

In order to examine the processes, we consider three cross sections in the system. Cut A-A lies directly in the axis of the fracture. Cut B-B has a distance of 2cm and C-C of 5cm from the fracture axis (see Figure 5). Figure 6 and Figure 7 show the water infiltration via fracture and matrix given by water saturation distributions at different time steps with the micro-scale and the meso-scale model concepts. The results show an overall very good agreement. However, (small) differences are caused by oscillations in the meso-scale model concept, for example in cut B-B, $t=30s$ (see Figure 7). These oscillations can probably be explained by the much coarser mesh resolution of the meso-scale model concept in the fracture-matrix surroundings (see the zooms in Figure 2 and 3).
FIGURE 6. Water saturation computed with the micro-scale model concept
FIGURE 7. Water saturation computed with the meso-scale model concept
5. CONCLUSIONS

In this paper, two model concepts for fractured porous media - so-called micro-scale and meso-scale model concept have been compared in a case study dealing with water infiltration into a domain with a vertical fracture. Overall, the model concepts agree well. Differences are caused by different mesh resolutions of the fracture-matrix surroundings. Future work includes the simulation of infiltration experiments carried out in a laboratory and the transition to the field scale.

Acknowledgements
We are grateful to the Land Berlin and the Deutsche Forschungsgemeinschaft who are for supporting this work within the NaFoG-Program and the Forschergruppe 581 'Großhang'.

References