ACCURATE MODELLING OF MATRIX-FRACTURE TRANSFERS IN FRACTURED POROUS MEDIA THROUGH SUB-GRIDDED DUAL-POROSITY MODELS

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Abstract

This paper proposes a new way to apply mixed models to the prediction of matrix-fracture exchanges in dual-porosity simulators. This model can be viewed as a sub-gridding technique, improving the reliability of exchange terms. First, this sub-gridding technique has been designed for the capillary imbibition case, by taking into account the physical specificities of this mechanism while minimizing the computational cost. Then it has been implemented in a conventional dual-porosity simulator. Moreover, fracture and matrix unknowns have been decoupled in order to reduce the CPU time. First results show the high accuracy of the simulated exchange terms, by comparison with conventional dual-porosity simulation results.

1. INTRODUCTION

Flows in porous media can be predicted with a satisfactory accuracy at the Darcy scale. At this scale, homogenized laws are available and flow equations are well-known. But the characteristic length-scales of many studied problems, such as petroleum reservoirs, prohibit the flow simulations at this local scale. Therefore, transfer mechanisms are upscaled at a higher scale, i.e. the reservoir simulation cell scale.

In naturally fractured petroleum reservoirs, two media are present: the matrix medium, which contains most of the oil reserves; and the fracture medium, which transport the hydrocarbons from the matrix medium to the wells. In the usual "dual-porosity" concept [Barenblatt *et al.*, 1960], [Warren and Root, 1963], flows in both media are upscaled to the reservoir simulation cell scale. For the matrix medium, fluid flows are often assumed to occur in a unique matrix block, supposed to be representative of all the set of matrix blocks composing a matrix simulation cell. All dual-porosity simulators use the same balance equations under the assumption of a pseudo-steady-state matrix-fracture exchange flow. Simulators may differ in the formulation of the exchange term, but as long as matrix-fracture transfer remains single-phase, involving only pressure diffusivity, the choice of one homogenization method or another has rarely any practical impact as the pressure equilibrium between fractures and matrix blocks is established almost instantaneously by comparison with the time accuracy required in reservoir engineering predictions, unless considering well testing cases. On the opposite, the choice of the homogenization method is crucial for multiphase flows.

Mixed models [Arbogast *et al.*, 1990], where exchanges take place between objects modelled at different scales, i.e. homogenized equations for the fracture network and Darcy-scale equations with mixed boundary conditions for the matrix blocks, avoid the dependence between the homogenized equations and the initial and boundary conditions of the studied case, which is the main problem of other methods, like the use of pseudo-functions [Thomas *et al.*, 1983]. This paper describes a mixed fractured reservoir model where a sub-gridding procedure of the matrix blocks provides a very accurate prediction of capillary-driven matrix -fracture transfers.

2. MODEL DESCRIPTION

The optimisation of the matrix blocks sub-gridding technique applied to capillary-driven mechanisms is based on the study of the kinetics of exchanges between the fracture and matrix media. This study is performed on a single matrix block, supposed to be representative of the set of matrix blocks in a given matrix simulation cell. This representative block is supposed to be initially saturated with a dead oil and irreducible water, and surrounded by fractures filled with water. To begin with, we consider an isotropic matrix block (in terms of permeability) having the same dimensions in each direction. The matrix rock is preferentially water-wet, and the matrix block height is small enough to allow us neglecting the impact of gravity forces on matrix-fracture exchanges. For our test cases, we choose the capillary pressure and relative permeability curves of the matrix medium as shown in Fig. 1. Cross-shaped curves are used for the fracture medium.

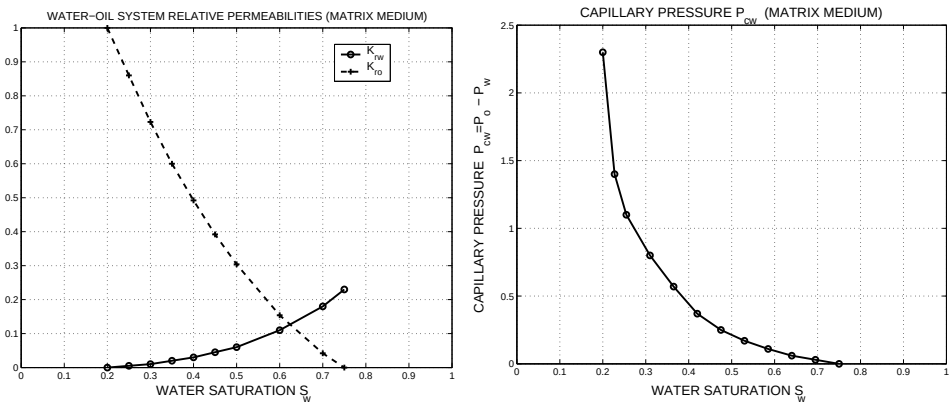

FIGURE 1. Relative permeabilities (*left*) and capillary pressure (*right*) curves in the matrix medium.

The mobility ratio is lower than one. In our simulation example, matrix porosity is 0.29 and matrix permeability is 1 md. In order to maintain fracture saturation constant during the simulations, fracture porosity is arbitrarily taken as very large. Fracture permeability is taken equal to 10 darcys. The boundary conditions on the matrix block surface are constant with time and corresponds to a full immersion in water.

2.1. **Sub-gridding optimisation.**

The sub-gridding optimisation is based on the study of the exchanges kinetics. The starting-point of the methodology is a reference fine-gridded simulation.

This reference simulation provides the evolution versus time of the normalized mean water saturation of the block, $\overline{S}^*(t) = \dfrac{\overline{S}(t) - S_{wi}}{1 - S_{orw} - S_{wi}}$. This curve is subdivided into intervals corresponding to constant mean saturation steps. For each step, the corresponding time t^n is denoted.

For each time t^n, the local saturation value $S^n(x)$ within the block is plotted versus the abscissa, taken from the block boundary. The resulting saturation profile, $S^n(x)$ is discretised into intervals corresponding to constant saturation steps, and the corresponding abscissae x_i^n are denoted. This set of abscissae $\{x_i^n\}$ constitutes an optimised discretisation of the saturation field within the block and, by extension, an optimal one-dimensional sub-grid of the block at that time t^n.

2.2. **Resulting optimised sub-grid.** The sub-grids previously obtained at various times t^n have to be lumped into a unique one, in order to create an optimised sub-grid which reproduces the matrix block behaviour at both short and long times. This is made by a simple algorithm, based on the imposed condition of strictly increasing cell dimensions from the fracture to the block center. In two- and three-dimensional flow cases, this procedure is applied in each flow direction, with the same number of sub-cells.

In order to validate the resulting sub-grid, a matrix-fracture capillary transfer was simulated on this optimized sub-grid and results were compared with those obtained on the reference highly-refined grid. It was found that the relative error does not exceed 1.7%, whatever the number of flow directions.

2.3. **Creation of an equivalent one-dimensional sub-grid.** In the case of capillary dominated flows, further model size reduction can be performed in order to reduce the numerical cost. Indeed, to describe exactly the physics of capillary imbibition phenomena, it is not necessary to reproduce the sub-grid in each direction (*cf.* Figure 2, *left*): a nested sub-grid (*cf.* Figure 2, *right*) can fairly well simulate the progression of the fluid front into the block. This nested sub-grid is obtained by supposing equal the unknowns (pressure and saturation) of all the grid cells composing a "ring".

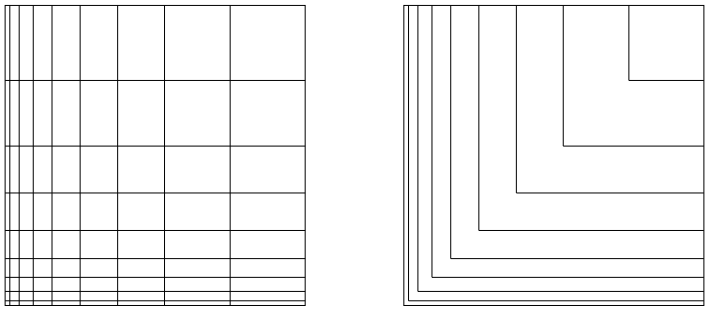

FIGURE 2. Two-dimensional case: optimised sub-grid reproduced in each direction (*left*) and nested sub-grid (*right*).

The equivalence between the two sub-grids is obtained by respecting four conditions:

- the number of "rings" is equal to the number of cells in a given flow direction of the multi-dimensional sub-grid.

- the total pore volume of the cells composing a "ring" is conserved.
- the "ring" porosity is equal to the average porosity within the cells composing a "ring".
- the global transmissivity between two adjacent "rings" is conserved.

The second condition implies the calculation of a new length of the cells and the choice of an arbitrary exchange area between cells.

2.4. **Validation of the equivalent one-dimensional model.** The equivalent one-dimensional model is validated by comparison with the reference model. A comparison is also made with the conventional dual-porosity model, in order to appreciate the improvement brought by the matrix block sub-gridding method. Simulations on the dual-porosity model have been made with the reservoir simulator ATHOS, with an option considering only capillary forces (and neglecting gravity and viscous effects). Fig. 3 compares the matrix oil recoveries predicted by the 3D-reference model (*SImbi_PMU_3D_8*, solid line), the 3D sub-gridded model (*SImbi_PMU_3D_opti3D*, dash-dotted line), the equivalent one-dimensional model (*SImbi_PMU_3D_opti1D*, dashed line) and the conventional dual-porosity model (*DImbi_PMU_3D_PC*, dotted line).

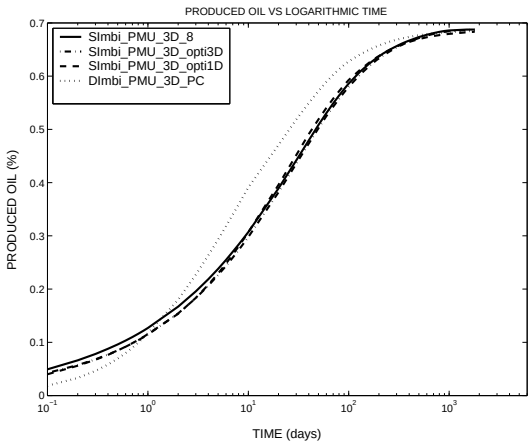

FIGURE 3. Prediction of the kinetics of capillary imbibition of the matrix block (3D case).

This kind of comparison was made for 1D, 2D and 3D cases and, in each case, results for the equivalent one-dimensional model were very close to the reference results. Fig. 3 also illustrates the improved accuracy of matrix-fracture exchanges calculated with the new sub-gridded model compared to the conventional dual-porosity model.

2.5. **Generalization and automation of the sub-gridding methodology.** The methodology previously presented can also be applied to anisotropic and non cubic blocks. The key idea is to use a substitution of variables to transform an anisotropic block into an isotropic one, with different dimensions. The direction of smallest length is then used in the sub-gridding methodology, and the resulting sub-grid is reproduced in each direction. The multi-dimensional sub-grid is then turned into a one-dimensional one, as explained before.

The influence of petrophysical parameters (capillary pressure and relative permeabilities curves and mobility ratio) has also been studied, and we can conclude that the optimised sub-grid does not differ very much with the shape or the petrophysical parameters of the block.

The required number of cells in the sub-grid was found to be always close to an average value of 14, with a mean progression ratio of the cells length comprised between 1.2 and 1.35. These general results allow the automation of the sub-gridding, by fixing the number of cells and the progression ratio between them. More details on this sub-gridding methodology and its validation can be found in [Famy *et al.*, 2005].

3. Implementation of the sub-gridded model in a reservoir simulator

Since the sub-gridding methodology provides accurate exchange terms, it has been implemented in a conventional dual-porosity flow simulator.

3.1. Conventional dual-porosity flow simulators.

Conventional flow simulators dedicated to fractured porous media are based on flow equations written at the simulation cell scale. These equations then refer to up-scaled flows.

For each phase, the equations in the two media are obtained by combining generalized Darcy's laws with mass conservation laws. The equations in both media are coupled by an exchange term, for each phase respectively.

In the capillary imbibition case, only two phases are present: water and oil. We neglect gravity effects and we suppose that each phase is composed by only one component. External source terms, represented by wells, are only present in the fracture medium. The resulting system can then be written as :

$$\left\{\begin{array}{rrrcl} \dfrac{\partial}{\partial t}(\phi^M \rho_w S_w^M) - div(K^M M_w^M grad\, P_w^M) & & -F_w & = & 0 \\ \dfrac{\partial}{\partial t}(\phi^F \rho_w S_w^F) - div(K^F M_w^F grad\, P_w^F) & -q_w^F & -F_w & = & 0 \\ \dfrac{\partial}{\partial t}(\phi^M \rho_o S_o^M) - div(K^M M_o^M grad\, P_o^M) & & -F_o & = & 0 \\ \dfrac{\partial}{\partial t}(\phi^F \rho_o S_o^F) - div(K^F M_o^F grad\, P_o^F) & -q_o^F & -F_o & = & 0 \end{array}\right. \tag{1}$$

where the superscript M (respectively F) refers to the matrix (respectively the fracture) medium, and the subscript w (respectively o) refers to the water (respectively oil) phase, and :

- $\phi = \phi(P)$ is the porosity at the local scale, multiplied by a volume fraction of matrix block,
- $\rho = \rho(P)$ is the density of the considered phase,
- S is the saturation of the considered phase,
- K is the permeability tensor of the considered medium,
- M is the mass mobility of the considered phase, defined as $M_p = \frac{k_{rp}\rho_p}{\mu_p}$, with $k_r = k_r(S)$ the relative permeability, and μ the viscosity,
- P is the pressure of the considered phase,
- q is the source term,
- F is the exchange term between the two media.

The two saturations S_w and S_o are related by $S_w + S_o = 1$, and P_w and P_o by the capillary pressure $Pc_w(S_w) = P_o - P_w$.

The calculation of the exchange terms F consists in calculating the exchange terms on a single matrix block, which is supposed to be representative of all the matrix blocks of the matrix simulation cell, and in multiplying this term by the number of matrix blocks on the cell.

It should be observed that the system (1) is highly non-linear, mainly due to the relative permeabilities, which can strongly vary between two adjacent cells. So the system is solved by using finite volumes, and the Newton's method. At each time step, the system is linearised with respect to pressure and saturation increments, and solved using a classical fully implicit or IMPES (IMplicit in Pressure, Explicit in Saturation) scheme [Aziz and Settari, 1979].

3.2. **The mixed model.** As shown in the previous chapter, the representative single matrix block is sub-gridded. Thus, matrix-fracture exchanges are calculated between the fracture cell and the first matrix block sub-cell, whereas they were previously calculated between the fracture cell and the homogenized matrix block (*cf.* Fig. 4, dashed arrows). Some additional exchanges must be calculated between the adjacent sub-cells in each block (*cf.* Fig. 4, solid arrows). Calculation of fracture-fracture exchanges are not modified (*cf.* Fig. 4, dash-dotted arrows).

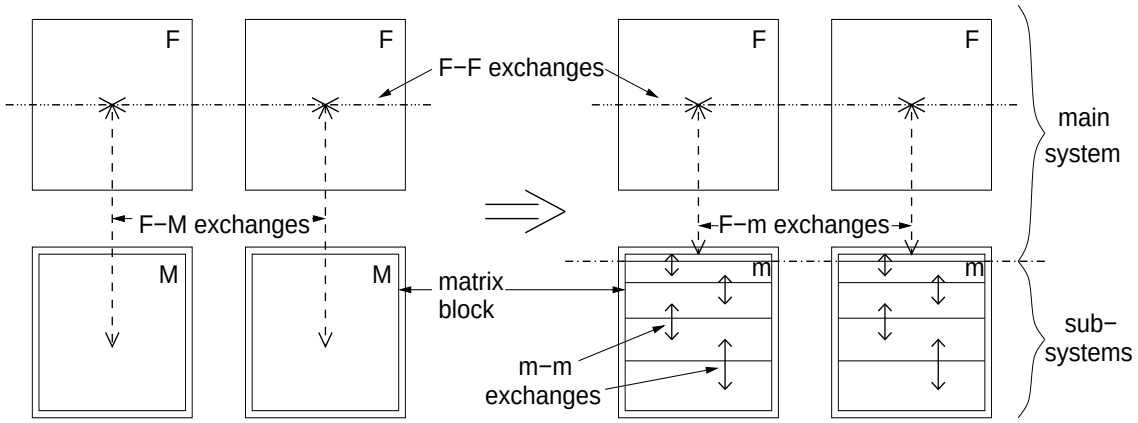

FIGURE 4. Matrix-fracture exchanges in the conventional dual-porosity case (*left*) and in the sub-gridded dual-porosity case (*right*).

The matrix-fracture exchange terms at the simulation cell scale are now formulated in our mixed model as follows:

$$F_p = N_b\, G(S_p^F)\, 2\, T^{m-F}_{I-1(I)} M^{m-F}_{p,I-1(I)} (P^F_{p,I} - P^m_{p,1(I)}) \tag{2}$$

where the subscript m refers to the sub-gridded matrix medium, $p = w$ or o, I is the number of the considered fracture cell, $1(I)$ is the first matrix sub-cell (which exchanges with fissure medium) corresponding to the fracture cell I and :

- N_b is the number of matrix blocks in a matrix simulation cell,
- $G(S_p^F)$ is a function that represents the number of smothered matrix blocks,
- V is the volume of the considered cell,

- $T^{m-F}_{I-1(I)}$ is the flow transmissivity between fracture cell I and the first sub-cell $1(I)$, defined by $T^{m-F}_{I-1(I)} = \frac{K^m A^m}{l_{1(I)}}$, with A^m the flow cross-section area of the block,
- M^{m-F} is the mobility at the interface between the two media, evaluated by an upstream scheme,
- $l_{1(I)}$ is the length between the center of the sub-grid $1(I)$ and the surface A^m.

Equations describing flow between matrix sub-cells are similar to equations of system (1), but without any exchange or source term, and are solved in the same way.

In order to minimize the extra computational cost, implied by the need to solve an additional matrix system, or sub-sytem, for each dual-medium cell, the unknowns of this matrix system are decoupled from those of the matrix-fracture system, or main system. Practically, the main system is solved first. This resolution provides the values of pressure and saturation in fracture cells and in the first sub-cells at the considered time step. After this, each matrix system is solved independently, by using the pressure and saturation values on the first sub-cells as Dirichlet boundary conditions. This decoupling is schematically represented on Fig. 4, on the right.

3.3. **First results.** The modified simulator has been tested first on a single matrix block, initially filled with oil. The sub-grid of the matrix block is composed by 15 sub-cells. The associated fracture cell is full of water. Fig. 6, left, compares the reference fine-grid (solid line) and the conventional dual-porosity (dotted line) simulation results with our mixed model: our method provides very accurate results, reproducing the reference solution.

The second test, sketched in Fig. 5, involves matrix block boundary conditions changing with time: the studied cell is surrounded by two cells, an upstream one containing a water injection well, and a downstream one containing a production well. No matrix-fracture exchanges take place in those well cells. All the cells are initially satured with oil, and water is injected from the beginning of the simulation.

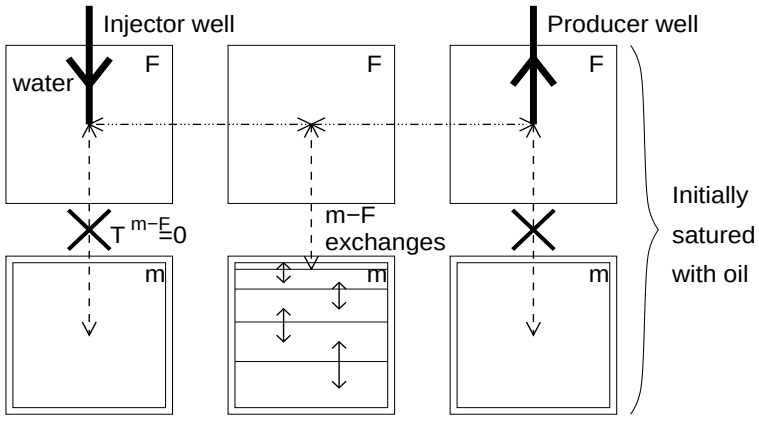

FIGURE 5. Second test case.

Once again, results obtained using the sub-gridded model are compared to a reference fine-grid and a conventional dual-porosity simulations. The results, shown in Fig. 6, on the right, are very promising. Additional tests are still in progress.

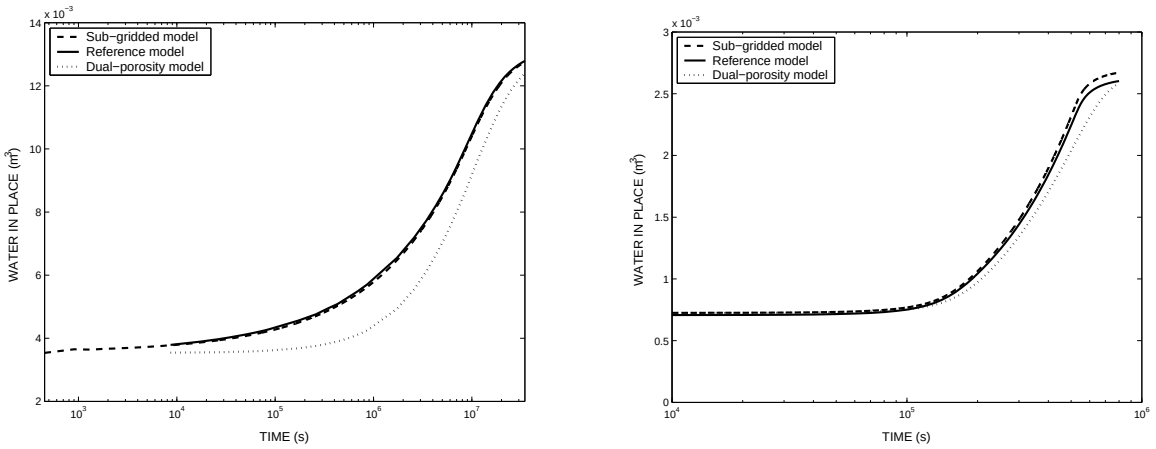

FIGURE 6. Water in place in the matrix block versus time: with fixed boundary conditions (*left*), and with non-stationnary block boundary conditions (*right*).

CONCLUSIONS

This study proposes a practical and innovative method to increase the reliability of matrix-fracture exchanges when simulating flows in porous fractured media. The approach consists in the application of a mixed model for which each fracture cell of a conventionnal dual-medium model is coupled with a detailed sub-gridded matrix block.

The methodology described here allows the creation of an optimised one-dimensional sub-grid for capillary-driven mechanisms, derived from the study of the local saturation evolution within the blocks.

The implementation of this mixed modelling approach in a conventional dual-porosity simulator led to very accurate exchange terms. It confirms the attractive perspectives of application of this methodology to water-driven fractured porous media where the matrix-fracture exchanges have a strong impact on multiphase flows.

Further extension of this methodology concerns more complex multi-phase matrix-fracture transfers, driven by gravity and capillary forces, for instance.

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