CROSS-GRADIENTS JOINT INVERSION OF DISPARATE GEOPHYSICAL DATA FOR IMPROVED SUBSURFACE CHARACTERISATION: MULTIPLE-PHYSICS FIELD EXAMPLES

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ABSTRACT

The characterisation and monitoring of hydrogeological and other complex subsurface processes demands the use of multiple geophysical data. Among the recently developed methodologies for data integration, it is the joint inversion based on the cross-gradient constraint that encourages models with geometrical similarity. In this work, I announce a generalised procedure to integrate multiple geophysical data in a procedure of joint inversion founded on the cross-gradient philosophy. I present two field data examples for near-surface environments that integrate P-wave, S-wave, DC resistivity and Audiomagnetotelluric data. In both examples, the multiple joint inversion procedure finds an underlying common structure and reveals new geophysical interrelationships particular for the studied sites.

1. INTRODUCTION

The characterisation and monitoring of hydrogeological and other complex subsurface processes requires a detailed knowledge of several properties of the composing rocks and fluids. Whilst some of these properties can be measured directly, other properties have to be estimated by indirect measurements such as geophysical data. However, it is not uncommon that the geophysical data yield models of limited accuracy that may not contribute significantly to our understanding of the subsurface processes.

Interestingly, the distribution of apparently uncorrelated physical properties seems to be controlled by common subsurface attributes that, when taken into account, can improve the accuracy and meaning of the otherwise independent geophysical models. For instance, a highly porous rock that is saturated with water can be well defined by its low seismic velocity and low electrical resistivity values. Many empirical relationships of this type are classical in the geophysical literature (e.g. *Archie*, 1942; *Gardner, et al.*, 1974; *Wyllie, et al.*, 1956) and other relationships are being intensively developed and implemented in joint inversion experiments (cf. *Berryman, et al.*, 2002; *Bosch and McGaughey*, 2001). However, such correlations are difficulty generalised to different geological environments. An outstanding feature of the subsurface that is common to the geophysical data is the geometrical distribution of the physical properties which can be measured by the physical property changes. This condition of commonality can be incorporated in the process of estimation to obtain meaningful and more reliable subsurface models in a process of joint inversion (e.g.

Gallardo and Meju, 2003; *Haber and Oldenburg*, 1997; *Saunders, et al.*, 2005; *Zhang and Morgan*, 1996). In particular, the cross-gradient approach (*Gallardo and Meju*, 2003) provides a flexible linking criterion that encourages the structural similarity between two images without assuming any parameter to parameter relationship.

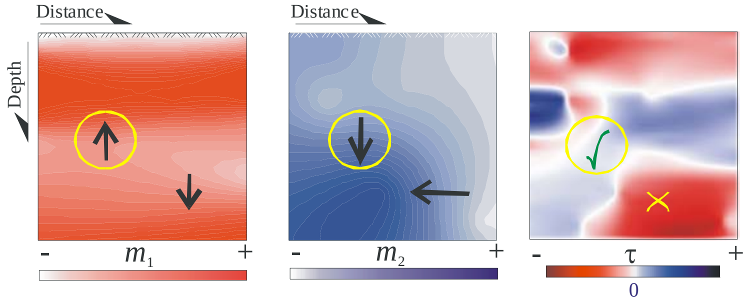
The beneficial effects of the cross-gradient philosophy for structural and lithological classification of two geophysical images are clearly documented in *Gallardo* (2004), *Gallardo and Meju* (2003; 2004) and *Gallardo, et al.* (2005). However, there is no reason to limit our expectations on subsurface characterisation to the perspective of just two geophysical data or parameters. We may instead ask if such amalgamated geophysical data will be beneficial to the estimation of geophysical parameters; or if the multiple geophysical data will just facilitate some petrophysical deductions or effectively reveal unknown petrophysical relationships. To help to elucidate some of these questions in this work, I announce, a generalized philosophy for multiple-physics parameter estimation based on the cross-gradient constraint and present some near-surface field examples that show the improvements attained in parameter accuracy, geometrical resemblance and their possible implications for geophysical and structural associations.

2. CROSS-GRADIENT PHILOSOPHY FOR JOINT INVERSION

A key issue in the search of structurally compatible models is the quantitative definition of the structural resemblance of two models. For this, I defined the cross-gradients function (τ) as the cross-product of the physical property gradients (*Gallardo and Meju*, 2003)

$$\vec{\tau}(m_1, m_2) = \nabla m_1 \times \nabla m_2 \,.$$

This function measures the structural similarity of two property images and the zero value of its magnitude defines the collinearity of two physical property changes as illustrated in Figure 1.



a) b) c)

FIGURE 1. Illustration of the concept of geometrical similarity between two images (a and b). The vectors represent the property gradients in corresponding zones. The circled vectors have meaningful amplitudes that point to opposite directions, implying structural similarity. The other illustrated vectors have significant amplitudes but point in different directions, implying no structural similarity. (c) Image of the corresponding values of the cross-gradient function. The largest positive or negative values of the cross-gradients are found in those areas with least structural similarity (after *Gallardo, et al.*, 2005).

The joint inversion is defined as the search of multiple models that satisfy their respective geophysical data in a least squares sense and are geometrically similar according to the cross-gradient values. Some a priori information, such as the smoothness assumption, is needed to account for those areas of the models that are not covered by the geophysical data. Using all this information, the final objective function is stated as:

$$\min \sum_i \left[misfit_i(m_i) + \phi_{apriori}(m_i) \right]$$

$$subject\ \ to\ \ \tau = 0$$

This objective function is the base to formulate algorithms for the joint inversion of DC resistivity, Magnetotelluric (MT) and Seismic travel time data. The objective function is minimized in an iterative process where the optimal model search is reduced to the space of geometrically similar models by the cross-gradient constraint, following a philosophy of solution similar to that described in *Gallardo and Meju* (2004).

3. STUDYING FULLY CORRELATED DATA: JOINT INVERSION OF MAGNETOTELLURIC, DC RESISTIVITY AND TRAVELTIME DATA

Aiming to provide a clearer understanding of the near surface materials in the area of Quorn in England, a research group from Leicester University established a pilot site that included a 200 m profile that was surveyed using several geophysical techniques (see *Meju, et al.*, 2003). The data collected included 13 Audio-magnetotelluric (AMT) soundings, 6 electrical soundings and a seismic refraction profile using five shot points. These geophysical data were interpreted separately by *Meju, et al.* (2003) and used to developed interesting resistivity-velocity correlations for the studied site. The same data have also been jointly inverted using cross-gradient constraints by *Gallardo and Meju* (2003; 2006). In this section the models are compared in terms of their contribution to a better integration of the three geophysical data.

Figure 2 shows the models obtained for two joint inversion experiments: the joint inversion of DC resistivity and seismic data (from *Gallardo and Meju*, 2003) and the joint inversion of AMT and seismic data (from *Gallardo and Meju*, 2006). Both experiments are important to gauge the cross-gradient methodology because the DC resistivity and the AMT data should provide a similar electrical image of the subsurface which should also be structurally identical to the common seismic image. Figure 2 illustrates the general success on this aim and Figure 3 confirms that the largest differences occurred in areas without simultaneous AMT and DC resistivity data coverage. Notably, the consistency of the models can be followed in the corresponding geophysical interrelationships (see Figure 4). This first example illustrates the convenience and consistency of the integration of multiple geophysical data that are basically sensitive to identical geophysical parameters. A different perspective using more than two geophysical parameters is explored in the following field example.

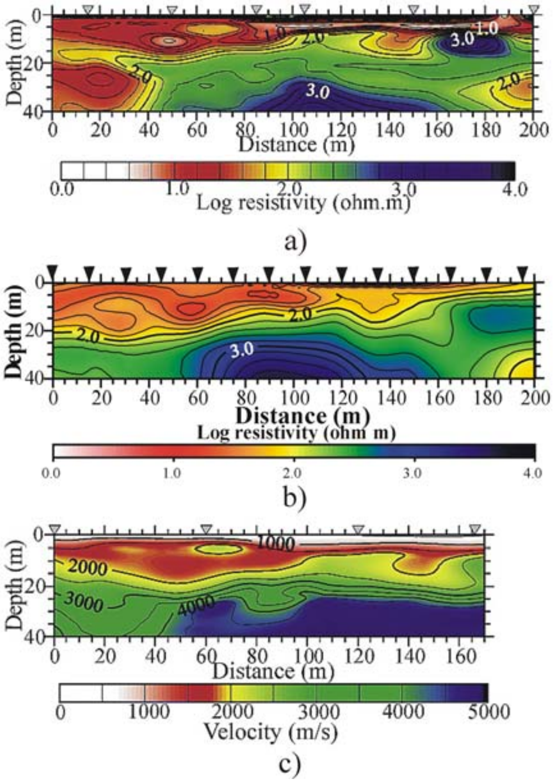


FIGURE 2. Resistivity and seismic images obtained from the joint inversion of DC-resistivity and seismic traveltimes (a and c) (After *Gallardo and Meju*, 2003) and from the joint inversion of AMT and the same seismic data (model b) (after *Gallardo and Meju*, 2006). The inverted triangles show either the sounding or the shot positions.

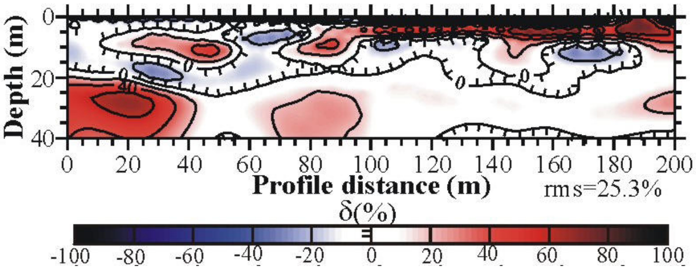


FIGURE 3. Differences between the two resistivity images illustrated in Figure 2. The maximum differences occur in areas without simultaneous data coverage.

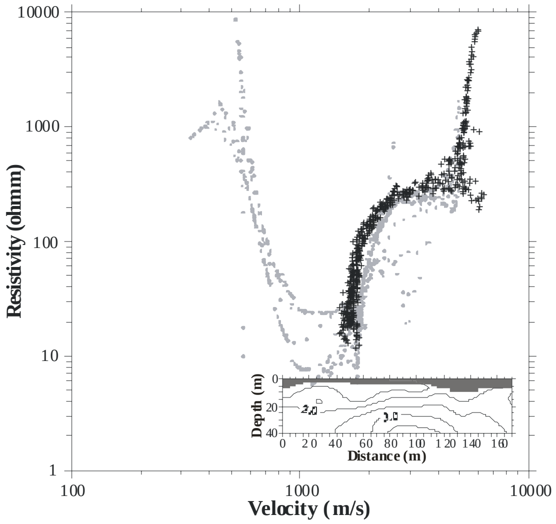


FIGURE 4. Comparison of the resistivity and seismic cross-plots for the studied site. The grey dots correspond to those obtained by joint inversion of DC-resistivity and seismic refraction data (*Gallardo and Meju*, 2003). The black crosses are extracted from the models resulting of the joint inversion of AMT-seismic data. The coincident trends correspond to the areas commonly covered by both data sets, which are illustrated in the inset (after *Gallardo and Meju*, 2006).

4. STUDYING UNCORRELATED DATA: MULTIPLE JOINT INVERSION OF P-WAVE, S-WAVE AND DC RESISTIVITY DATA

This test site was established over a buried hillside. The primary objective of this survey is to explore the advantages of multiple-physics estimation for the characterisation of a typical sedimentary basin produced by alluvial and residual deposits at the flanks of volcanic ranges. Similar sites are common sources of water for local settlements in the west coast of Baja California. The studied profile includes conventional P- and SH-wave seismic refraction travel times registered by vertical and horizontal geophones. The profile also includes DC resistivity data for 6 vertical soundings and a Dipole-Dipole profile as illustrated in Figure 5.

To set a comparative framework, the geophysical data were inverted separately, producing the Vp, Vs and resistivity property images depicted in Figures 5a, 5c and 5e. For the joint inversion experiment, the P-wave, S-wave and DC resistivity data are jointly inverted to produce their corresponding geophysical images (illustrated in Figures 5b, 5d and 5f). Similarly to the previous joint inversion experiments, the only assumed correlating factor is that given by the cross-gradient constraint.

Figure 5 shows that, whereas the images derived from the separate inversion suggest some common structures, the images that result from joint inversion are geometrically similar. As a result of the agreement of the three geophysical data, the images should evince more closely the true underlying geological structure. In analogy to previous experiments, the hypothesis of a common structural behaviour should also help to discern the parameter to parameter relationships for the subsurface materials, which can be the base for petrophysical deductions.

Figure 6 shows how the jointly inverted data enhance the parameter to parameter correlation and furnish a wider perspective for multiple geophysical correlations. For instance, The Vp/Vs correlation found for jointly inverted data is remarkably clear in the corresponding cross-plot. The correlations between resistivity and seismic parameters reveal, on the other hand, a more complex interrelationship, which was expected for both physical phenomena.

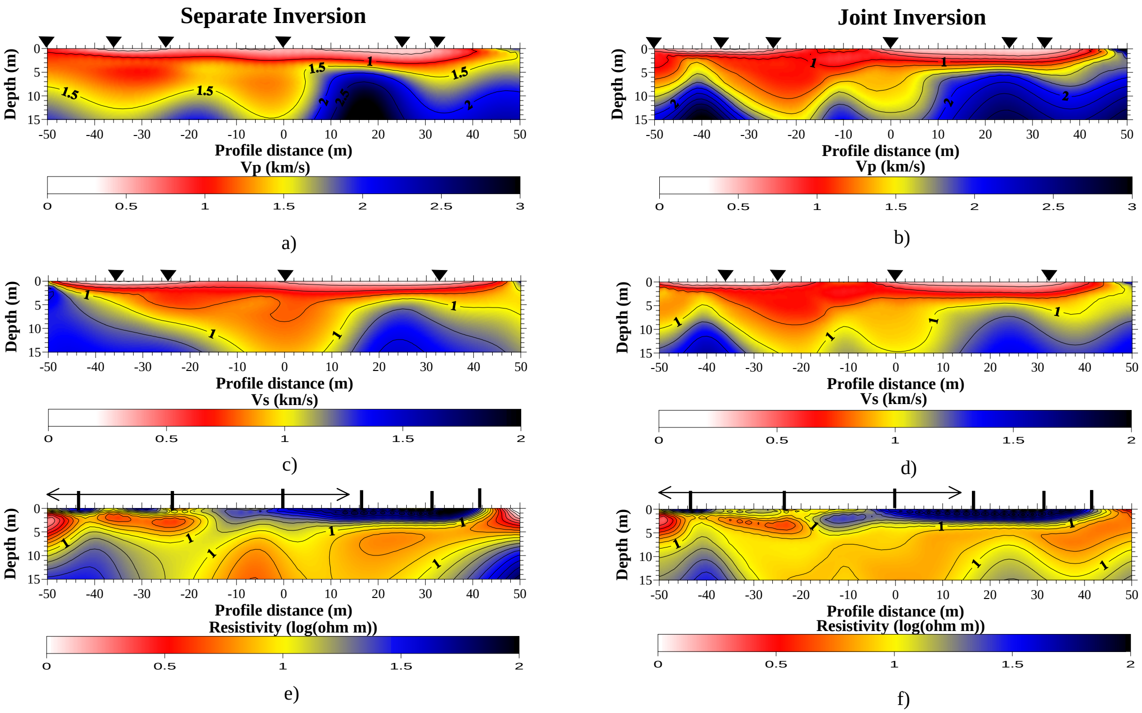


FIGURE 5. Images obtained from the separate inversion (a, c, and e) and multiple joint inversion (b, d and f) of P-wave travel times, SH-wave travel times and DC resistivity data. In all the figures the inverted triangles denote the shot locations. The rectangles mark the sounding positions and the line, the extension of the dipole-dipole profile.

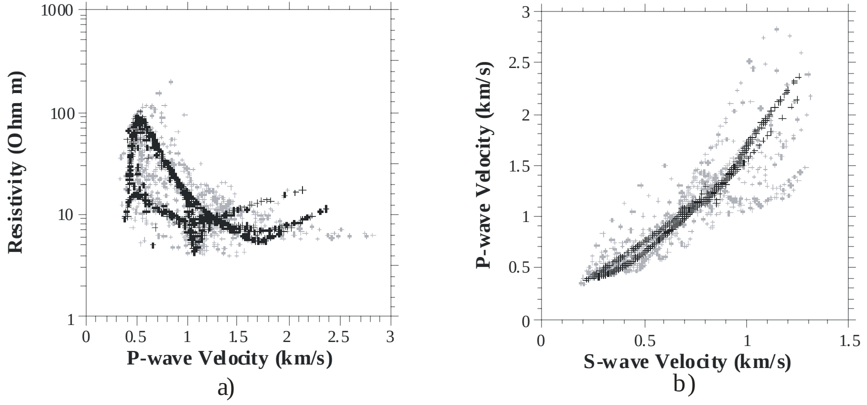


FIGURE 6. Cross-plots of the resistivity and seismic velocity values obtained from separate inversion (grey crosses) and joint inversion (black crosses) of P-wave, SH-wave and DC resistivity data

5. CONCLUSIONS

The joint inversion of multiple geophysical parameters using cross-gradient constraints offers a robust procedure to elucidate subsurface features and processes through structurally consistent images of multiple physical parameters. The field examples in particular, showed the robustness of the methodology when facing data redundancy and the existence of an underlying structure common to three types of geophysical data. The examples also illustrate the consistency of the geophysical relationships found and reveal the more complex interrelationships that occur in real multi-physics environments that characterise the rocks and fluids in typical near-surface environments.

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