THRESHOLD SCALES FOR SPATIALLY AVERAGED SOIL MOISTURE AND EVAPOTRANSPIRATION WITH RAINFALL HETEROGENEITY

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ABSTRACT

An outstanding issue in ecohydrologic modeling is scaling nonlinear plant-level interactions among soil, vegetation, and water to larger spatial scales. Spatial heterogeneity in precipitation and vegetation exert significant control on scaling properties, especially in water-limited ecosystems. Recent investigations have demonstrated that relationships between variables controlling soil-moisture dynamics are hysteretic at larger averaging scales, even when unique relationships are imposed at the plant level. The evolution of these relationships with increasing averaging area depends on the properties of the ecosystem. The goal of this investigation is to identify a threshold scale for spatially averaged soil moisture and evapotranspiration beyond which the characteristics of the non-unique relationships will vary only slowly as averaging area becomes larger. The threshold scale, which is analogous to the concept of a representative elementary area (REA), enables identification of a large-scale relationship that is meaningful with respect to ecosystem properties. We focus on the case of spatial and temporal heterogeneity of rainfall due to thunderstorms in Texas. Simulation results demonstrate that, if vegetation is homogeneous, REAs likely exist at scales between 10 and 100 times the mean size of storm rain cells. However, if heterogeneous vegetation cover occurs, then a threshold scale is not reached before violating the assumption of a stationary rainfall process for the Texas thunderstorm case.

1. INTRODUCTION

The dynamic linkage among soil, plants, and atmosphere through the temporal and spatial variability of soil moisture is a fundamental component of ecohydrological models [*Henderson-Sellers*, 1996; *Rodriguez-Iturbe, et al.*, 1999]. A critical issue for efforts to improve model predictions of land-surface processes is the disparity between the resolution of land-surface models and the spatial scale at which physical relationships controlling soil-moisture dynamics are known [*Crow and Wood*, 2002]. That is, when plant-scale relationships are nonlinear and heterogeneity in land-surface properties and variables exists, then it is generally inappropriate to apply the plant-scale relationships at large spatial scales [*Crow and Wood*, 2002; *Nordbotten, et al.*, In press; *Puma, et al.*, submitted]. Typically, spatial heterogeneity exists at scales smaller than the resolution of these models, which results in hysteretic relationships among spatially averaged variables that control soil-moisture dynamics [*Puma, et al.*, submitted].

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The characteristics of the non-unique relationships depend on a model's resolution (that is, the size of the averaging area) and the spatial properties of the soil, vegetation, and rainfall [*Puma, et al.*, *submitted*]. As averaging area increases, the characteristics of the non-unique relationships evolve. The objective of this investigation is to determine whether a threshold area can be determined, beyond which the characteristics of the hysteretic relationships would vary only slowly as averaging area becomes larger. This threshold scale is analogous to the concept of a representative elementary area (REA), which has been the focus of multiple investigations and discussions in the literature [*Bloschl, et al.*, 1995; *Famiglietti and Wood*, 1995; *Fan and Bras*, 1995; *Wood, et al.*, 1988].

2. METHODOLOGY

Soil-moisture dynamics are simulated using a mosaic approach, which involves discretization of the land surface into homogeneous grid blocks that directly and independently exchange fluxes with the atmosphere. We analyze a relatively flat, water-limited ecosystem, so that we do not need to use a catchment-based approach for identification of a threshold scale [e.g., *Famiglietti and Wood*, 1995; *Wood, et al.*, 1988]. Instead, we take a simpler approach that uses square averaging areas with coinciding centroids at the center of the modelled area. The spatial averages will be an area or volume average depending on the physical interpretation of each variable. In particular, we look to identify an REA for the case of spatially heterogeneous precipitation due to thunderstorms, assuming that rainfall is a stationary random process. We also assess the effect of vegetation cover on REA values.

2.1 Water balance at the plant scale.
The model used to simulate soil-moisture dynamics in each grid block of the mosaic is based on a soil-water balance at a point and is expressed as [*Laio, et al.*, 2001b]

$$\phi^i Z_r^i \frac{dS^i(t)}{dt} = I^i(t) - Q^i\left[S^i(t), t\right] - ET^i\left[S^i(t)\right] - L^i\left[S^i(t)\right] \qquad (1)$$

where $S(t)$ is the relative soil moisture content ($0 \leq S(t) \leq 1$), ϕ is porosity, Z_r is the depth of the root zone [mm], $I(t)$ is the infiltrating rainfall rate [mm/d], $Q[S(t),t]$ is the runoff rate [mm/d], $ET[S(t)]$ is the evapotranspiration rate [mm/d], $L[S(t)]$ is the leakage rate [mm/d], and t is time. Superscript i denotes the grid block. We assume that the root zones are non-overlapping in space and that they do not interact. For simplicity of notation, we will not indicate the time dependence of soil moisture in what follows unless it is necessary. We evaluate Equation (1) using the analytical expressions derived by *Laio et al.* [2001b] for soil-moisture decay during inter-storm periods.

2.2 Temporal and spatial rainfall model
Rainfall input is treated as an external random forcing that is independent of soil moisture. The occurrence of storm events is modeled as a series of point events in time that arise according to a Poisson process with rate λ_t [e.g., *Laio, et al.*, 2001b]. That is, we ignore the temporal structure within storm events and apply them as a Dirac delta functions in time. In space, a storm event is composed of spatially varying pulses of rainfall that represent daily precipitation. The spatial distribution of rainfall for a storm event is based on a simplified statistical description of the cellular structure of a storm event following *Rodriguez-Iturbe et al.* [1986]. Each storm event is represented by a collection of rain-producing cells with each cell characterized by total depth of rainfall at the cell center and by a spread function, which specifies the decay of rainfall depth with distance from the cell center [*Rodriguez-Iturbe, et*

al., 1986]. The cell centers are distributed over the region R in a two-dimensional Poisson process of density λ_{xy}.

We obtain the depth of rainfall at a cell center from an exponential distribution with mean rainfall depth $E[h]$. The total rain depth deposited in grid block i (Y^i) is the sum of contributions from all cells in the region

$$Y^i = \sum_{(x^j, y^j) \in R} h^j \cdot g(r) \tag{2}$$

where h^j is the rainfall depth at cell center j due to cell j, $g(r)$ is a spread function, r is the distance between (x^i, y^i), the center of grid block i, and (x^j, y^j), the center of the rain cell j. We model the spread of rainfall around cell centers according to a quadratic exponential function [*Rodriguez-Iturbe, et al.*, 1986]

$$g(r) = \exp\left(-2\left(\frac{r}{a'}\right)^2\right) \tag{3}$$

where a' [km] represents a characteristic spatial scale of a rain cell. The moments of this model are [*Rodriguez-Iturbe, et al.*, 1986]

$$E[Y] = \frac{1}{2} \cdot \pi \lambda_{xy} (a')^2 E[h] \tag{4}$$

$$\text{Var}[Y] = \frac{1}{2} \cdot \pi \lambda_{xy} (a')^2 E^2[h] \tag{5}$$

For the simulations in this paper, we assume that a' is known, and we obtain $E[Y]$ and $\text{Var}[Y]$ from rainfall data. We then solve for λ_{xy} and $E[h]$ by combining Equations (4) and (5).

2.3 Interception, runoff, evapotranspiration, and leakage.

Canopy interception of rainfall is modeled following the simplified approach of *Laio et al.* [2001b] by setting a fixed threshold depth, Δ, for each vegetation type. If a simulated storm produces a rainfall depth less than the fixed depth in a given grid block, then all of the rainfall is intercepted. Infiltrating rainfall will be the total rainfall minus Δ for rainfall depths greater than Δ. Runoff is given by

$$Q^i\left[S^i(t), t\right] = \max\left[0, Y^i - \phi^i Z_r^i\left(1 - S^i(t^-)\right)\right] \tag{6}$$

where $S^i(t^-)$ is the relative soil-moisture content at an instant before time t in grid block i. That is, runoff only occurs if the water-storage capacity of the soil is exceeded; there is no Hortonian (infiltration excess) mechanism for runoff.

A piecewise-linear function is used to approximate the plant-scale evapotranspiration relationship as [e.g., *Laio, et al.*, 2001b; e.g., *Rodriguez-Iturbe, et al.*, 1999]

$$ET^i(S^i) = \begin{cases} E_w^i \dfrac{S^i - S_h^i}{S_w^i - S_h^i} & S_h^i < S^i \leq S_w^i \\[2mm] E_w^i + \left(ET_{\max}^i - E_w^i\right)\dfrac{S^i - S_w^i}{S^{*,i} - S_w^i} & S_w^i < S^i \leq S^{*,i} \\[2mm] ET_{\max}^i & S^* < S^i \leq 1 \end{cases} \tag{7}$$

where S_h is the relative soil-moisture content at the hygroscopic point, S_w is the relative soil-moisture content at wilting, S^* is the relative soil-moisture content at incipient stomatal closure, E_w is the evaporation rate at S_w, $ET_{\max}$ is the maximum evapotranspiration rate, and superscript i denotes the grid block.

The leakage from the root zone is modeled as [*Laio, et al.*, 2001b]

$$L^i\left(S^i\right) = \frac{K_s^i}{e^{\left(2b^i+4\right)\left(1-S_{fc}^i\right)} - 1}\left[e^{\left(2b^i+4\right)\left(S^i-S_{fc}^i\right)} - 1\right], \quad S_{fc}^i < S^i \le 1 \qquad (8)$$

where K_s is the saturated hydraulic conductivity [mm/d], b is an experimentally determined parameter, and S_{fc} is the soil moisture at field capacity. Leakage is zero for soil moisture values less that field capacity.

2.4   Model parameters for Texas savanna.

The ecosystem examined in this analysis is a savanna parkland/woodland vegetation complex in the *La Copita Research Area* (27°40'N, 98°12'W) that has been extensively studied by the Texas Agriculture Experiment Station. The land surface is characterized by a flat landscape with very mild slopes that is covered by a diphase tree-grass vegetation [*Laio, et al.*, 2001a; *Scifres and Koerth*, 1987]. Although the ecosystem's potential natural vegetation is classified as *Prosopis-Acacia-Andropogon-Setaria* savanna, a shift from grass to woody plant domination has occurred within the past 150 years [e.g., *Archer*, 1995]. *Prosopis glandulosa* (honey mesquite) is the dominant woody plant [e.g., *Archer*, 1995; *Archer, et al.*, 1988; *Brown and Archer*, 1990; e.g., *Scifres and Koerth*, 1987], which coexists with C_4 grasses including *Paspalum setaceum* [*Archer, et al.*, 1988]. The properties of individual *Prosopis glandulosa* and *Paspalum setaceum* are presented in Table 1.

TABLE 1.   Parameters values for the woody plant *Prosopis glandulosa* and the C_4 grass *Paspalum setaceum* in a sandy loam in Texas from *Laio et al.* [2001a], based on data from *Hass and Dodd* [1972], *Cuomo et al.* [1992], *Ludlow* [1976], *Stroh et al.* [1996], and *Wan and Sosebee* [1991].

	S_w	$S*$	Z_r [mm]	ET_{max} [mm/d]	E_w [mm/d]	Δ [mm]
Prosopis glandulosa	0.180	0.350	1000	4.42	0.20	2.0
Paspalum setaceum	0.167	0.370	400	4.76	0.13	1.0

The eastern Rio Grande Plains has approximately 70% of its annual rainfall occurring from April to September [*Laio, et al.*, 2001a; *Scifres and Koerth*, 1987]. We assign a value to the characteristic size of a rain cell (as captured by parameter a') that is consistent with observation of thunderstorms [*Morin, et al.*, In press; *Sorman*, 1975]. Daily precipitation records from the National Climatic Data Center station at Benavides, Texas are used to estimate $E[Y]$ and $Var[Y]$. For this analysis, we consider only the period from May 15 to June 16 to obtain a rainfall record that is approximately homogeneous (statistically) in time, because temporal rainfall statistics change during the growing season [*Rodriguez-Iturbe, et al.*, 1987a; *Rodriguez-Iturbe, et al.*, 1987b; *Rodriguez-Iturbe, et al.*, 1984]. λ_{xy} and $E[h]$ are then calculated using Equations (4) and (5). These parameters are given in Table 2.

TABLE 2.   Parameter values characterizing the temporal and spatial characteristics of thunderstorms during the growing season in eastern Rio Grande Plains of Texas.

Parameter	Value
Storm arrival rate, λ_t	0.167 d^{-1}
Mean rainfall depth at cell center, $E[h]$	25.2 mm
Number of cells per area, λ_{xy}	0.0155 cells/km^2
Mean rainfall depth in grid block i, $E[Y^i]$	15 mm
Storm cell parameter, a'	5 km

The soil in La Copita is spatially heterogeneous. Approximately 57 percent of the site has an A horizon of fine sandy loam, which is uniformly distributed over the area [*Scifres and Koerth*, 1987]. To keep the focus on spatial heterogeneity in rainfall and vegetation, we make the simplification that the soil of the root zone is homogeneous. Table 3 presents the soil parameters.

TABLE 3.   Soil parameters values for sandy loam in Texas from *Laio et al.* [2001a], based on data from *US DOA* [1979] and *Clapp and Hornberger* [1978].

Parameter	Value
Saturated conductivity, K_{sat}	822 mm/d
Porosity, ϕ	0.43
Soil retention parameter, b	4.9
Relative soil-moisture at field capacity, S_{fc}	0.56
Relative soil-moisture at hygroscopic point, S_h	0.14

We simulate three land-surface types, a grassland, a savanna, and a woodland. For the savanna, the vegetation is a matrix of grass with woody plants located according to a Poisson process in space [e.g. *Rodriguez-Iturbe, et al.*, *In press*]. The woody plants are assumed to have circular crowns, where the radius of each crown is obtained from an exponential distribution. We discretize the land surface into 5×5 m grid blocks, which have an area approximately equal to the average canopy area of the woody species *Prosopis glandulosa*. For simulation purposes, the soil-moisture balance is calculated for a sparse sample of grid blocks, which has been verified to reproduce predictions of spatially averaged variables from the full sample.

3. RESULTS AND DISCUSSION

The rainfall model simulates thunderstorms, which are characterized by spatial heterogeneity of rainfall produced through the number and spatial location of rain cells and the distribution of rain within these cells. The heterogeneity is also a function of time because of the random number, location, and depth of rain cells. As a first attempt to generalize our results with respect to this heterogeneity, we divide averaging area, A, by the mean size of a rain cell within a storm, $E[A_{rain\_cell}]$, which is equal to 130 km^2 for our simulations.

The scale-dependent, non-unique relationships discussed by Puma [*submitted*] between spatially averaged evapotranspiration, $<ET>$, and soil moisture, $<S>$, are presented in Figure 1 for a hypothetical *Paspalum setaceum* grassland. Each marker on the plot corresponds to a single set of daily $<S>$ and $<ET>$ values. A non-unique relationship exists between the spatially averaged variables, even though the unique relationships from Equation (7) are applied locally (solid line). The characteristics of the non-unique relationships are scale-dependent, as demonstrated by observing the change in the distribution of data when $A/E[A_{rain\_cell}]$ increases from 0.5 to 5.0 (Figures 1a and 1b).

To identify a threshold scale for the relationship between $<ET>$ and $<S>$, we first compute $<S>$ versus $A/E[A_{rain\_cell}]$ and $<ET>$ versus $A/E[A_{rain\_cell}]$ for 200 realizations. We plot six of the realizations, which are representative of the range of relationships that we see for a grassland (Figures 2a and 2b). Figure 2 shows that $<S>$ and $<ET>$ values for small averaging areas can be significantly different from the values over a larger area for the same realization. This case could arise if the small averaging area received either significantly more or less rainfall than the surrounding region over some previous time. For most

realizations, we find that fluctuations in $<S>$ and $<ET>$ values decrease considerably as $A/\mathrm{E}[A_{\mathrm{rain\_cell}}]$ increases.

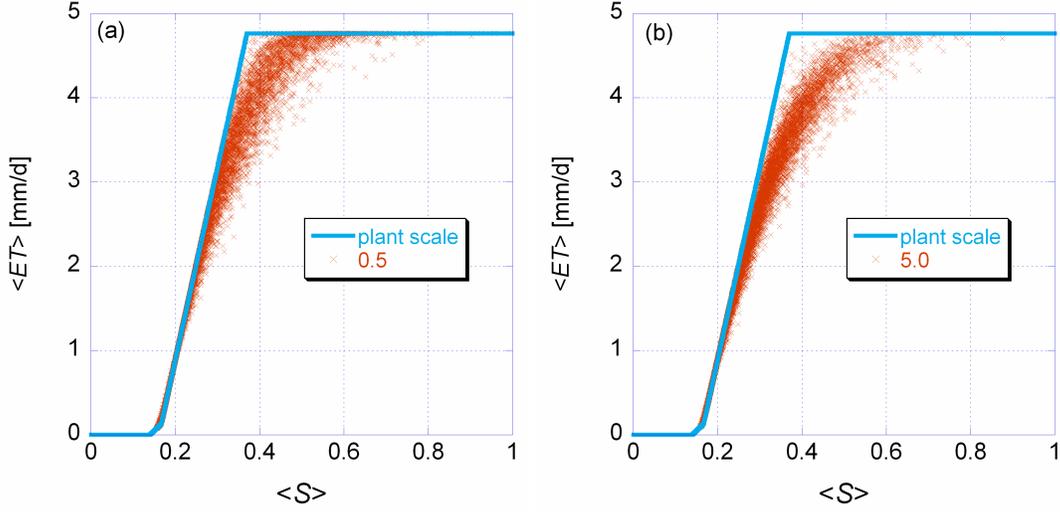
FIGURE 1. Relationships between $<ET>$ versus $<S>$ as predicted by the mosaic model when rainfall is spatially non-uniform in a hypothetical Texas grassland compared with the plant-scale function for $A/\mathrm{E}[A_{\mathrm{rain\_cell}}]$ equal to (a) 0.5 and (b) 5.0.
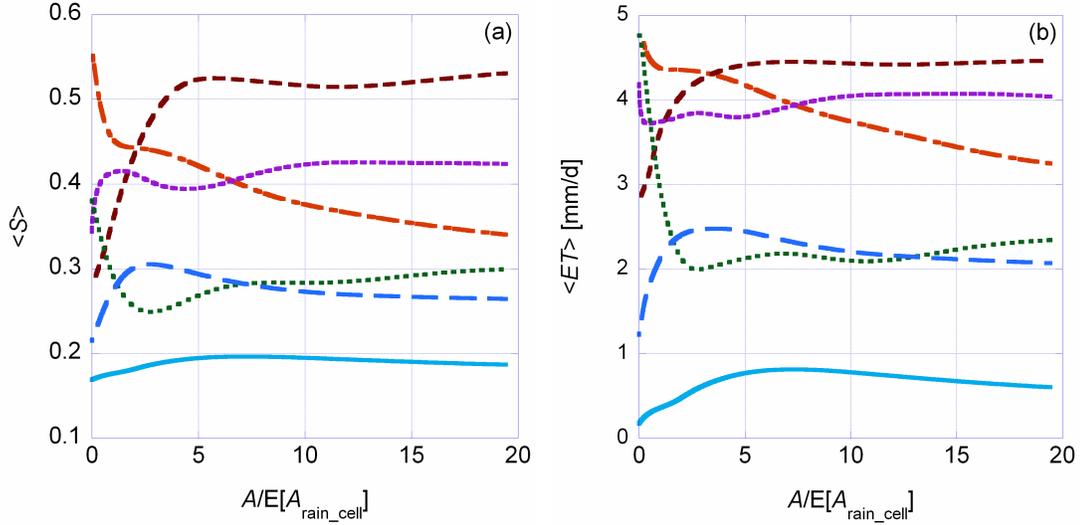

FIGURE 2. Spatially averaged (a) soil moisture and (b) evapotranspiration as functions of $A/\mathrm{E}[A_{\mathrm{rain\_cell}}]$ for multiple realizations.

We next calculate the variance across the realizations for $<S>$ and $<ET>$ and plot these versus $A/\mathrm{E}[A_{\mathrm{rain\_cell}}]$ as shown in Figure 3. This variance will approach a constant value as the fluctuations in $<S>$ and $<ET>$ diminish with increasing $A/\mathrm{E}[A_{\mathrm{rain\_cell}}]$ (as in Figure 2). The constant variance implies that the averaging area is large enough, such that $<S>$ and $<ET>$ values will not change significantly with increasing averaging area. This relates to the $<ET>$ versus $<S>$ relationships shown in Figure 1, because it implies that the distribution of data

will not shift significantly with further increase in averaging area. That is, a threshold scale or REA has been reached when the variance across realizations becomes constant, and the relationship between $<ET>$ and $<S>$ can then be considered a stable, large-scale relationship.
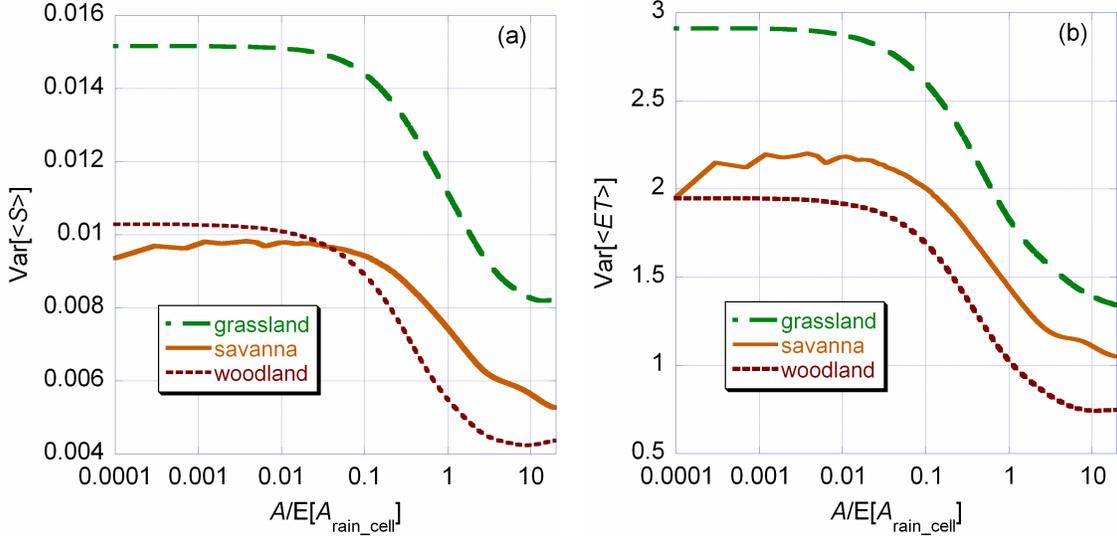

FIGURE 3. Variance of (a) $<S>$ and (b) $<ET>$ across 200 realizations as a function of $A/\mathrm{E}[A_{\mathrm{rain\_cell}}]$ for hypothetical grassland, savanna, and woodland ecosystems in Texas.

Figure 3 shows the results for three types of vegetation cover. The grassland and woodland results reveal that the effects of rainfall heterogeneity are evident for values of $A/\mathrm{E}[A_{\mathrm{rain\_cell}}]$ larger than 0.01. In the case of the savanna, vegetation heterogeneity produces fluctuations in the variance of $<S>$ and $<ET>$ with change in averaging area for $A/\mathrm{E}[A_{\mathrm{rain\_cell}}]$ values smaller than 0.01. For the range of $A/\mathrm{E}[A_{\mathrm{rain\_cell}}]$ values between 0.01 and 10, the variances of the spatially averaged variables decrease significantly as $A/\mathrm{E}[A_{\mathrm{rain\_cell}}]$ becomes larger. Although it appears that the grassland might have reached a threshold scale for $<S>$ after $A/\mathrm{E}[A_{\mathrm{rain\_cell}}]$ reaches a value of 10, Figure 3b demonstrates that a threshold scale has not been reached, because the variance of $<ET>$ is still decreasing as $A/\mathrm{E}[A_{\mathrm{rain\_cell}}]$ gets larger. The woodland might have reached an REA. However, additional simulations must verify this inference, since the variance of $<S>$ has a slight upward slope for $A/\mathrm{E}[A_{\mathrm{rain\_cell}}]$ larger than 10. Finally, the savanna clearly does not reach an REA as demonstrated by both the variance of $<S>$ and $<ET>$ versus $A/\mathrm{E}[A_{\mathrm{rain\_cell}}]$.

It is important to highlight the assumption that rainfall is a stationary random process. That is, the rainfall model is only valid for regions where the rainfall statistics do not change over time or space. This assumption becomes less appropriate for averaging areas that are larger than what has been presented herein.

4. CONCLUSIONS

For the case of spatial and temporal heterogeneity of rainfall due to thunderstorms in Texas, simulation results demonstrate that, if vegetation is homogeneous, REAs likely exist at scales between 10 and 100 times the mean size of storm rain cells. If heterogeneous vegetation cover occurs, then a threshold scale is not reached before violating the assumption of a stationary rainfall process for the Texas thunderstorm case.

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