TWO DIMENSIONAL HYDROLOGICAL SIMULATION IN ELASTIC SWELLING/SHRINKING PEAT SOILS

MATTEO CAMPORESE[1], CLAUDIO PANICONI[2], MARIO PUTTI[3], PAOLO SALANDIN[1], PIETRO TEATINI[3]

[1]Dept. of Hydraulic, Maritime, Environmental, and Geotechnical Engineering, University of Padova, Italy.
[2]Centre Eau, Terre et Environnement, Institut national de la recherche scientifique, Université du Québec, Canada.
[3]Dept. of Mathematical Methods and Models for Scientific Applications, University of Padova, Italy.

Abstract

Peat soils respond to drying/wetting cycles due to evapotranspiration and precipitation with reversible deformations induced by variations of water content. This process results in short-term vertical displacements of the soil surface and induces variations in the peat hydraulic properties that cannot be neglected when dealing with water flow problems in peatlands. A constitutive model describing swelling/shrinkage dynamics in peat soils under unsaturated conditions is implemented in a finite element simulator of Richards' equation. The model entails a significant modification of the general storage capacity term. The contribution of the saturated zone to the total deformation is taken into account, using the theory of primary consolidation in the hypotheses of completely reversible volume changes and constant compressibility. Simulations have been carried out for a drained cropped peatland south of the Venice Lagoon (Italy), for which a large data set of hydrological and displacement measurements has been collected since the end of 2001. The considered domain is a field section bounded by ditches, subject to rainfall, evapotranspiration, and lateral drainage. The comparison between simulated and measured quantities demonstrates the capability of the model to accurately reproduce both the hydrological and deformation dynamics of peat.

1. INTRODUCTION

Because of its high compressibility and organic matter content, peat soils respond to natural hydrologic cycles of wetting and drying with reversible deformations due to variations of water content [Ingram, 1983]. Two different responses to pressure changes can be identified: compression (expansion) below the water table due to the high compressibility of the bulk, and shrinkage (swelling) above the water table due to water content variations and the hydrophilic character of the soil organic fraction [Kennedy and Price, 2004]. Depending on the ratio between the thickness of the unsaturated and saturated zones, shrinkage and compression may have different relative importance, even though it is recognized that the rate of volume change in the unsaturated zone is greatest [Kennedy and Price, 2004; Price, 2003]. Both processes manifest themselves as short-term vertical displacements of the soil surface: fibrous, poorly decomposed peatlands may experience displacements of the order of 10 cm, whereas surface movements are relatively small (≈1 cm) in highly mineralized amorphous organic soils [Deverel and Rojstaczer, 1996; Schlotzhauer and Price, 1999].

The sequence of swelling and shrinkage events, known also as "mire breathing", is a key issue for the hydrologic behavior of peatlands, since it produces short-term changes in the pore structure, density, and hydraulic properties of peat. Saturated hydraulic conductivity, retention curves, water storage, and porosity in particular must be considered dynamically variable with water content [Kennedy and Price, 2004]. These variations exert a strong influence on water flow and storage in peatlands, with important implications for peatland hydrology and ecology, in particular with respect to restoration [Price, 2003].

In order to reproduce real dynamics of peat response to soil water content variations, several swelling/shrinking models of peat soils have been proposed. Some of these models are empirical [Oleszczuk *et al.*, 2003; Hendriks, 2004], and require both fitting and/or site specific measured parameters. On the other hand, physically based models [Pyatt and John, 1989; Price, 2003; Kennedy and Price, 2004] often require a large number of measurable parameters, but adequate calibration data from laboratory experiments and in situ tests are not always available.

Starting from the approach of Pyatt and John [1989], valid only for saturated conditions, we apply a two-parameter constitutive model that relates the short-term reversible dynamics of the peat surface to the hydrologic characteristics of the unsaturated zone. The peat soil moisture is linked to the void ratio, which in turn affects the thickness of the peat layer. This relationship is used to obtain an adapted version of Richards' equation, in which the porosity is allowed to vary dynamically with saturation. Porosity evolution calculated from the flow equation solution allows the determination of the soil surface displacements, these being the integral of deformations over the total thickness of the aquifer. This approach has been implemented in a finite element code that has been applied to a peat experimental site located south of the Venice Lagoon, Italy [Gambolati *et al.*, 2005]. The comparison of the model results with field measurements over a period of three months containing a number of rainfall events supports the validity of the proposed approach.

2. THE MODEL

2.1. **The constitutive relationship.** Swelling/shrinkage of peat is described by constitutive relationships relating volume variations to moisture content changes. A volume V of peat is considered, expressed as $V = V_s + V_v$, where V_s is the volume of the solid fraction and V_v the volume of the voids. The voids can be partially or totally filled by water, and hence the water volume fraction V_w can be smaller or at most equal to V_v. Let e be the void ratio, defined as V_v/V_s, and ϑ the moisture ratio, equal to V_w/V_s and linked to the volumetric water content $\theta = V_w/V$ by the function $\vartheta = \theta(1+e)$. According to Camporese *et al.* [2006], the constitutive relationship relating void ratio to moisture ratio for peat soils is expressed as:

$$e = \begin{cases} (\vartheta_0 + 1)^{1-\delta}(\vartheta + 1)^{\delta} - 1 & \text{if } \vartheta \leq \vartheta_0 \\ \vartheta & \text{if } \vartheta > \vartheta_0 \end{cases}, \tag{1}$$

where ϑ_0 is the threshold moisture ratio above which the soil is fully saturated, and the exponent δ characterizes the deformation dynamics. If $\delta = 1/3$ the deformations are isotropic, while $\delta \neq 1/3$ define anisotropic deformation patterns. For $\delta = 1$ the volume changes occur only along the vertical direction. Equation (1) is valid under the following two hypotheses: i) the soil is not subject to cracks, i.e., it can always be considered as a continuum, implying that $1/3 < \delta \leq 1$, and ii) the deformations are elastic, i.e., completely reversible.

2.2. **Adaptation of Richards' equation.** To take into account reversible deformations while modeling water flow, relationship (1) must be included in the flow equation for variably saturated porous media, also known as Richards' equation, that can be written in terms of pressure head ψ as:

$$\sigma \frac{\partial \psi}{\partial t} = \nabla \cdot [K_s K_r \nabla(\psi + z)] + q, \tag{2}$$

where $\sigma(S_w)$ is the general storage term, with $S_w(\psi) = V_w/V_v$ being the water saturation, t is time, ∇ is the spatial gradient operator, K_s is the saturated hydraulic conductivity, $K_r(S_w)$ is the relative hydraulic conductivity, z is the vertical coordinate (positive upward), and q represents a source or sink term. The equation is completed by appropriate boundary and initial conditions, and by the retention curves $S_w(\psi)$ and $K_r(S_w)$, which can be given following any of numerous relationships available in the literature [e.g., van Genuchten, 1980]. The general storage term is commonly given as:

$$\sigma = S_w S_s + \phi \frac{\partial S_w}{\partial \psi}, \tag{3}$$

where S_s is the specific elastic storage coefficient that takes into account the elastic compressibility of the porous matrix and $\phi = V_v/V$ is the porosity of the medium. When dealing with mineral soils in unsaturated flow conditions, the term $S_w S_s$ can be often neglected as compared to $\phi \partial S_w/\partial \psi$, whereas in saturated conditions $\partial S_w/\partial \psi$ vanishes and S_s becomes important. In organic soils however, where volume changes due to soil moisture variations need to be taken into account, porosity varies with space and time as a function of water saturation and the elastic storage coefficient may not be negligible [Schlotzhauer and Price, 1999]. Thus, volume changes in the unsaturated zone are taken into account by incorporating equation (1) into Richards' equation, yielding the following expression for σ [Camporese *et al.*, 2006]:

$$\sigma = \frac{e}{1+e} \frac{\partial S_w}{\partial \psi} \left[\frac{\delta (e S_w + 1)^{-1}}{1/S_w - (\vartheta_0 + 1)^{1-\delta} \delta (e S_w + 1)^{\delta - 1}} + 1 \right]. \tag{4}$$

Equation (4) is valid only in unsaturated conditions, while for fully saturated soils $\sigma = S_s$.

2.3. **Numerical formulation.** Richards' equation with the modified general storage term given by (4) has been implemented in a three-dimensional finite element (FEM) simulator called FLOW3D [Paniconi and Putti, 1994], one of the modules of CATHY (CATchment HYdrology), a coupled model for the numerical simulation of surface and subsurface flow [Putti and Paniconi, 2004]. The nonlinear system of equations, which arises at each time step from the temporal and spatial discretization, is solved using the Picard technique: linearization is achieved by evaluation of the nonlinear terms at the previous iteration level [Paniconi and Putti, 1994]. Because of the nonlinear form of (4), its actual value is updated within each Picard iteration as well [Camporese *et al.*, 2004; Camporese, 2005].

For each node of the discretization and each time step, the vertical deformation of the soil layer (defined as the ratio between current and initial thickness) is computed as [Camporese *et al.*, 2006]:

$$\frac{\ell(t)}{\ell(0)} = \left[\frac{1+e(t)}{1+e(0)}\right]^{\delta}, \tag{5}$$

where $t = 0$ refers to the initial stage. The time evolution of the void ratio is evaluated solving equation (1) in the unsaturated zone. When $S_w = 1$ the vertical deformation is evaluated following the theory of soil consolidation as reported in Gambolati [1973]. Being p the water pressure, we assume that the edometric compressibility of the bulk is constant and given by:

$$C_b = \frac{1}{1+e}\frac{de}{dp}, \tag{6}$$

so that straightforward integration yields

$$e(p) = (1+e(0))\, exp(C_b p) - 1. \tag{7}$$

Defined as γ_w the specific weight of water and β its compressibility, we thus assume a constant elastic storage coefficient given by $S_s = \gamma_w(C_b + \phi\beta) \simeq \gamma_w C_b$, being $C_b \gg \phi\beta$. This value for $e(t)$ is then used in (5) with $\delta = 1$ to obtain the actual vertical deformation associated with the saturated layer.

3. APPLICATION

The model performance has been evaluated by comparison with a large measured data set collected at the Zennare Basin, a drained cropped peatland south of the Venice Lagoon, in which an experimental project is in progress since the end of 2001 [Gambolati *et al.*, 2005]. Subsidence measurements collected hourly by a properly designed instrument [Camporese *et al.*, 2006] show that significant reversible peat volume changes are caused by variation of soil moisture. Peat swelling and shrinkage have been observed after every rainfall event, with a very rapid swelling dynamics after precipitation, followed by shrinkage progressing at a slower rate. The overall process closely resembles the water table behavior and exhibits a time scale ranging between a few hours and a few weeks [Gambolati *et al.*, 2005].

3.1. **Simulation set-up.**

3.1.1. *Model domain.* A vertical section representative of the experimental plot conditions has been used to simulate the measured displacements and hydrological data. The profile of the section is shown in Figure 1, together with the location of the different instruments installed in the field. The domain surface has a slope of 0.005 and is discretized by 3 rows of 54 right-angled triangles each, disposed to obtain a ≈14 m long grid of unit width. The surface triangles are then replicated vertically 16 times to form 15 layers, yielding a three-dimensional mesh of 7290 tetrahedral finite elements and 1792 nodes, with a total thickness of the peat stratum varying between 1.20 on the left boundary and 1.27 m on the right boundary.

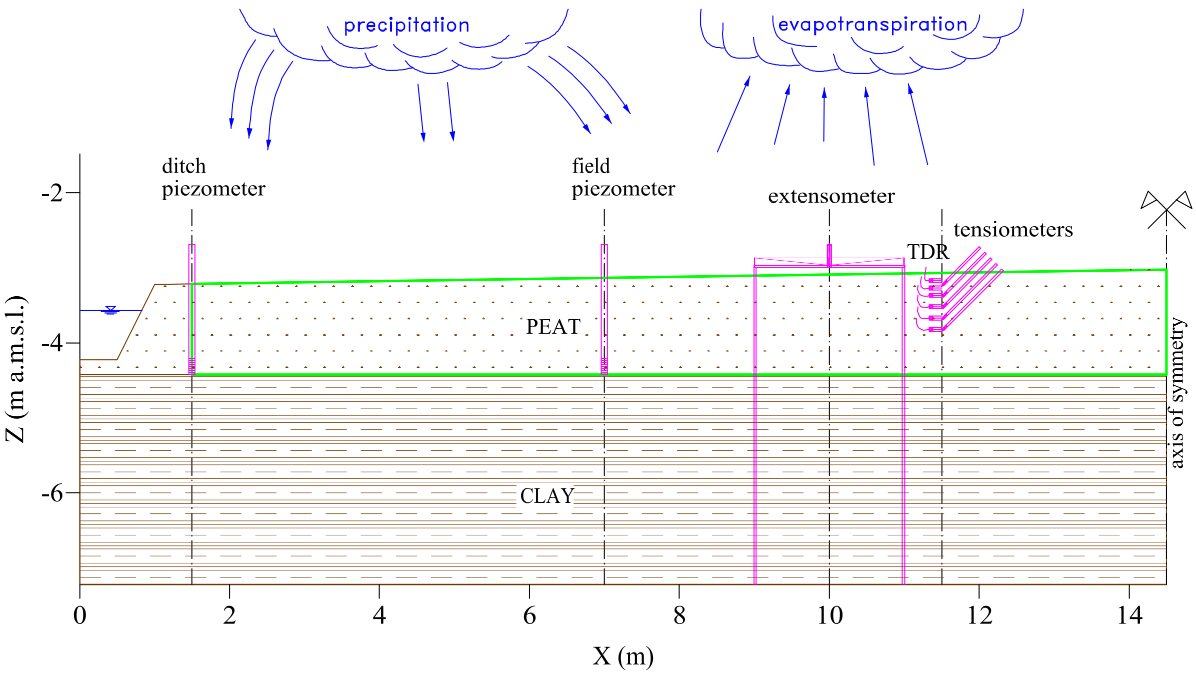

FIGURE 1. Profile of the considered section and location of the different instruments placed in the field

3.1.2. *Boundary and initial conditions.* Boundary conditions are imposed as follows. The base of the domain is assumed impermeable, since the peat layer is bounded by a thick low-conductivity clay formation. Rainfall rates available from rain gage measurements and potential evapotranspiration estimated by the FAO-Penman-Monteith formula [Allen *et al.*, 1998] are applied on the surface. The water table on the left-hand side lateral boundary is imposed to vary according to the water levels in the adjacent ditch, while the right-hand side is considered as a no-flux boundary, because of symmetry considerations. Seepage face boundary conditions are imposed above the water table on the left-hand side of the mesh.

Initial conditions are represented by a partially saturated vertical hydrostatic profile based on the recorded piezometer level in the middle of the experimental plot. The first period of 200 hours is used to dissipate the uncertainty on the initial conditions [Camporese *et al.*, 2006].

3.1.3. *Parameterization.* The integration of Richards' equation (2) with the general storage term expressed by (4) requires the determination of the retention curves $S_w(\psi)$ and $K_r(S_w)$, the constitutive model parameters ϑ_0 and δ, the saturated hydraulic conductivity K_s, and the elastic storage coefficient S_s. The retention curves are described by the Van Genuchten model [van Genuchten, 1980]. Estimated values of θ_r, ψ_s, and n (0.45, -0.37 m, and 1.76, respectively) are evaluated from experimental retention curves determined by relating θ and ψ measurements collected at Zennare by means of TDR probes and tensiometers [Camporese *et al.*, 2006]. The same experimental data allow the assessment of the threshold moisture ratio ϑ_0, that varies with depth from a minimum value of

1.32 at the soil surface to a maximum value equal to 9.00 at the peat bottom. Also the parameter δ increases with depth, because of the increased overburden load [Oleszczuk *et al.*, 2003]. Camporese *et al.* [2006] have shown that for the Zennare peatland δ changes from 0.35 at the surface to 0.50 at the bottom of the considered domain. The vertical and horizontal components of the saturated hydraulic conductivity tensor have been estimated in Camporese [2005] to be equal to 5×10^{-7} m/s and 7×10^{-5} m/s, respectively. Finally, the specific elastic storage coefficient S_s has been taken from Schlotzhauer and Price [1999] and fixed at 2.6×10^{-2} m^{-1}. Note that all the parameters have been derived from field measurements or calibrated by previous applications carried out on different time intervals [Camporese, 2005].

3.2. **Simulation results.** The model has been applied over the period between 05 August and 28 October 2005 that was characterized by several significant rainfall events, alternated by periods of high evapotranspiration. Figure 2 reports the comparison between the simulation results and the corresponding measurements collected at the Zennare Basin during the considered time interval. The agreement between the water table measured by the field piezometer and simulated by the model is generally good, especially during the last rainfall event. On the other hand, a few peaks in the soil surface elevation are slightly underestimated by the model. The agreement between experimental and computed pressure head values is satisfactory, while simulated water content does not match measured values, even if the dynamics is quite similar between 1000 and 1500 hours. This occurrence may be due to the significant hysteresis that has been observed in the retention curves during the considered period [Camporese, 2005] that is not taken into account by the model.

4. CONCLUSIONS

A constitutive model for reversible peat deformations related to soil moisture variations in the unsaturated zone has been implemented in Richards' equation through a suitable modification of the general storage term that allows consideration of porosity changes with water saturation. A finite element code based on this modified formulation has been applied to a large data set collected in an experimental site located south of the Venice Lagoon. The application of the model by means of a two-dimensional field section shows generally a satisfactory agreement between measured and simulated soil surface displacements and pressure heads, while the accuracy of the simulated water content is much lower, probably due to inadequate characterization of the retention curves.

Acknowledgments

This work has been partially supported by Co.Ri.La., Consorzio di Bonifica Adige-Bacchiglione, Municipalities of Cavarzere, Chioggia and Cona (Venice-Italy), Servizio Informativo del Magistrato alle Acque per la Laguna di Venezia, and the University of Padova project entitled "Multiscale monitoring of CO_2 fluxes from agricultural soils and modeling of the spatial variability of the sources for quantification and control of emission into the atmosphere".

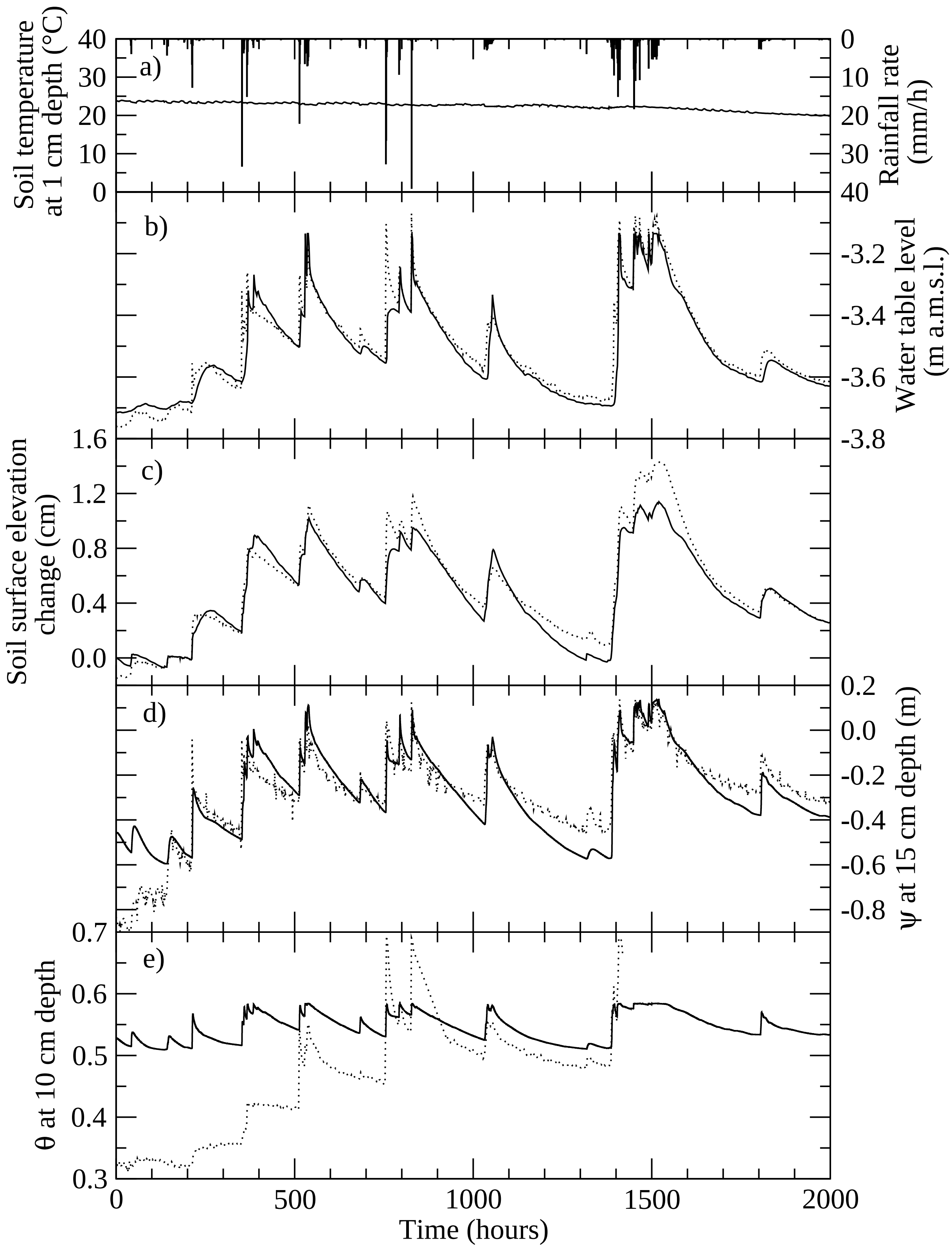

FIGURE 2. Simulation of a field section bounded by ditches. a) Rainfall rate (vertical bars) and soil temperature at 1 cm depth (solid line) measured in the Zennare basin. b-e) Comparison between simulated data (solid lines) and measurements in the middle of the experimental plot (dotted lines): b) water table level, c) soil surface elevation change, d) pressure head at 15 cm depth, and e) volumetric water content at 10 cm depth.

References

Allen, R. G., Pereira, L. S., Raes, D., and Smith, M. (1998). Crop evapotranspiration - guidelines for computing crop water requirements. Irrigation and drainage paper 56, FAO, Rome.

Camporese, M. (2005). *Modeling and experimental analysis of peat hydrology and its relationships with the subsidence of the Zennare Basin, Venice (Italy)*. Ph.D. thesis, Università degli Studi di Padova.

Camporese, M., Putti, M., Salandin, P., and Teatini, P. (2004). Modeling peatland hydrology and related elastic deformation. In C. T. Miller, M. W. Farthing, W. G. Gray, and G. F. Pinder, editors, *Computational Methods in Water Resources*, volume 2, pages 1453–1465, Amsterdam. Elsevier.

Camporese, M., Ferraris, S., Putti, M., Salandin, P., and Teatini, P. (2006). Hydrological modeling in swelling/shrinkage peat soils. *Water Resour. Res.* In press.

Deverel, S. J. and Rojstaczer, S. (1996). Subsidence of agricultural lands in the Sacramento-San Joaquin Delta, California: Role of aqueous and gaseous carbon fluxes. *Water Resour. Res.*, **32**(8), 2359–2367.

Gambolati, G. (1973). Equation for One-Dimensional Vertical Flow of Groundwater 1. The Rigorous Theory. *Water Resour. Res.*, **9**(4), 1022–1028.

Gambolati, G., Putti, M., Teatini, P., Camporese, M., Ferraris, S., Gasparetto Stori, G., Nicoletti, V., Rizzetto, F., Silvestri, S., and Tosi, L. (2005). Peatland oxidation enhances subsidence in the Venice watershed. *Eos Trans. AGU*, **86**(23), 217–224.

Hendriks, R. F. A. (2004). An analytical equation for describing the shrinkage characteristic of peat soils. In J. Päivänen, editor, *Wise Use of Peatlands, Proceedings of the 12th International Peat Congress*, pages 1343–1348, Jyväskylä, Finland. Int. Peat Society.

Ingram, H. A. P. (1983). Hydrology. In A. J. P. Gore, editor, *Mires: Swamp, Bog, Fen, and Moor*, volume 4A of *General Studies*, pages 67–158. Elsevier Sci., New York.

Kennedy, G. W. and Price, J. S. (2004). Simulating soil water dynamics in a cutover bog. *Water Resour. Res.*, **40**, W12410, doi:10.1029/2004WR003099.

Oleszczuk, R., Bohne, K., Szatylowicz, J., Brandyk, T., and Gnatowski, T. (2003). Influence of load on shrinkage behavior of peat soils. *J. Plant Nutr. Soil Sci.*, **166**, 220–224.

Paniconi, C. and Putti, M. (1994). A comparison of Picard and Newton iteration in the numerical solution of multidimensional variably saturated flow problems. *Water Resour. Res.*, **30**(12), 3357–3374.

Price, J. S. (2003). Role and character of seasonal peat soil deformation on the hydrology of undisturbed and cutover peatlands. *Water Resour. Res.*, **39**(9), 1241, doi:10.1029/2002WR001302.

Putti, M. and Paniconi, C. (2004). Time step and stability control for a coupled model of surface and subsurface flow. In C. T. Miller, M. W. Farthing, W. G. Gray, and G. F. Pinder, editors, *Computational Methods in Water Resources*, volume 1, pages 1391–1402, Amsterdam. Elsevier.

Pyatt, D. G. and John, A. L. (1989). Modelling volume changes in peat under conifer plantations. *Journal of Soil Science*, **40**, 695–706.

Schlotzhauer, S. M. and Price, J. S. (1999). Soil water flow dynamics in a managed cutover peat field, Quebec: Field and laboratory investigations. *Water Resour. Res.*, **35**(12), 3675–3683.

van Genuchten, M. T. (1980). A closed-form equation for predicting the hydraulic conductivity of unsaturated soils. *Soil Sci. Soc. Amer. J.*, **44**, 892–898.