PATH-AVERAGE RAINFALL ESTIMATION USING A MICROWAVE LINK: UNCERTAINTY DUE TO RAINFALL SPATIAL VARIABILITY

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Abstract

Microwave links offer the possibility to estimate (1) the path-average evaporation along the link, as well as (2) the path-average rainfall intensity along the link when precipitation occurs. In the latter case, a power-law relation is generally assumed between the specific attenuation k affecting the electro-magnetic signal while propagating through precipitation and the rainfall intensity R, variable of interest for many hydrological applications. This work focuses on the influence of the spatial variability of rainfall along the link on the parameterization of the R-k relation, at the point and link scale, for wavelengths ranging from 0.3 up to 3 cm. A stochastic model is used to generate range profiles of raindrop size distributions and therefore to simulate link measurements. A Monte Carlo approach allows to determine the optimal wavelength for a microwave link with respect to rain rate estimation (found to be about 0.9 cm), and to quantitatively analyze the uncertainty associated with path-average rain rate estimates, for different types of rainfall events.

1. INTRODUCTION

Single-frequency microwave links offer the opportunity to simultaneously estimate path-average evaporation and precipitation along their paths (*Leijnse et al.*, 2006a,b). A link typically covers a range of a few kilometers, with a transmitter on one end and a receiver at the other, and at a height well inside the surface layer of the atmosphere.

During dry periods, the sensible (temperature-related) and latent (humidity-related) heat fluxes, associated with turbulent eddies, cause the refractive index to fluctuate. These fluctuations in turn cause in the link signal to scintillate. The scintillations mainly depend on the sensible heat flux in the optical domain, while they depend on both fluxes in the microwave domain. Therefore, additional information is needed to estimate both fluxes from microwave link scintillations (e.g. an energy budget constraint, as proposed by *Leijnse et al.* 2006a).

During rainy periods, the microwave link signal is attenuated by the raindrops along its path. Microwave links take advantage of the fact that over a certain range of frequencies, the specific attenuation k is almost linearly related to the rainfall intensity R. Consequently, the path-average rainfall intensity $< R >$ is proportional to the (one-way) path-average specific attenuation $< k >$. The path-average specific attenuation itself is proportional to the (one-way) path integrated attenuation (PIA) which is measured by the receiver of the link. The proportionality factor between $< R >$ and $< k >$ depends on the rain drop size distribution (DSD) along the path. Because the DSD is highly variable in space (and time), it is important to investigate the influence of the DSD on the $< R > - < k >$ relation, in terms of variability along the link and between rain events.

In this paper, we use a stochastic simulator to generate range profiles of DSDs, from which all the microwave link variables can be derived. A Monte Carlo approach enables to quantitatively analyze the impact of the DSD variability on the accuracy of the precipitation estimates as performed by a microwave link, for a range of wavelengths and for different types of rain events. Section 2 presents a brief description of the stochastic DSD simulator. Section 3 is devoted to the analysis of the influence of the link wavelength on the coefficients of the $< R > - < k >$ relation. In Section 4, the uncertainty on the path-average rain rate estimated by a microwave link is quantified. Finally, in Section 5 we present the conclusions of this study.

2. DSD SIMULATOR

The range profiles of DSDs have been generated using the stochastic simulator proposed by *Berne and Uijlenhoet* (2005). It is based on an exponential DSD model:

$$N(D|N_t, \lambda) = N_t \, \lambda \, e^{-\lambda D} \ , \tag{1}$$

where $N(D|N_t, \lambda)dD$ denotes the drop concentration in the diameter interval [D,D+dD] given N_t (drop concentration in m^{-3}) and λ (in mm^{-1}). The two parameters N_t and λ are assumed to be random variables, jointly lognormally distributed. To introduce a spatial structure in the profiles, $N' = \ln N_t$ and $\lambda' = \ln \lambda$ are assumed to follow a first order discrete vector auto-regressive process. This results in an exponential auto-correlation function:

$$\rho(r) = e^{-2r/\theta} \ , \tag{2}$$

where r represents the distance lag and θ the characteristic spatial scale, also known as the scale of fluctuation:

$$\theta = 2 \int_0^{+\infty} \rho(r) \, dr \ . \tag{3}$$

The stochastic simulator is able to produce range profiles of DSDs of equivolumetric spherical drops.

DSD time series measurements from an optical spectropluviometer, collected during the HIRE'98 experiment (*Uijlenhoet et al.*, 1999) in Marseille, France, are used to parameterize the simulator for two types of rainfall. A first set of parameters is derived from the measurements of the entire 7 September 1998 rain event, and corresponds to moderate rainfall intensities (about 30 mm h^{-1} on average). A second set of parameters is derived from a 45-min period of intense rainfall (about 60 mm h^{-1} on average), and corresponds to convective rainfall. Assuming Taylor's hypothesis with a constant velocity of 12.5 m s^{-1}, consistent with the wind speed estimate of *Berne et al.* (2004), the required spatial characteristics of N' and λ' are derived. To achieve a high spatial resolution of 100 m, DSD data have been analyzed at a 8-s time step. The length of the link (hence of the simulated profiles) is fixed to 5 km. The analysis of the fitted N' and λ' values shows that the cross-correlation is negligible. The number of model parameters now reduces to five: the mean and standard deviation of N' and λ', and the characteristic scale θ (assumed to be equal for N' and λ'). Their values are given in Table 1.

TABLE 1. Mean, standard deviation and characteristic time of $N' = \ln N_t$ and $\Lambda' = \ln \lambda$ deduced from HIRE'98 data (entire and 45-min intense rainfall period of the 07/09/1998 event) at a 8 s time step.

		Mean	Std	θ (m)
N'	entire	7.77	0.46	6450
	intense	7.95	0.50	3600
λ'	entire	1.05	0.20	6450
	intense	0.85	0.31	3600

A total number of 400 profiles of DSD parameters have been generated for a range of wavelengths between 0.3 and 3 cm. From these DSD profiles and at each wavelength, the corresponding profiles of (one-way) specific attenuation k and rain rate R have been computed, using the Mie theory for the scattering cross-sections (*van de Hulst*, 1981) and Beard's velocity model for the drop terminal fall velocity (*Beard*, 1977). The specific attenuation (in dB km^{-1}) is defined as

$$k = \frac{1}{\ln 10} \int_0^\infty \sigma_E(D) N(D|N_t, \lambda)\, dD \tag{4}$$

where σ_E [cm^2] is the extinction cross-section. Similarly, the rain rate (in mm h^{-1}) is defined as

$$R = 6\pi\, 10^{-4} \int_0^\infty D^3 v(D) N(D|N_t, \lambda)\, dD \tag{5}$$

where $v(D)$ denotes the terminal fall speed (in m s^{-1}) of a drop of diameter D. Fig. 1 shows an example of a generated profile of specific attenuation k and rain rate R, at a wavelength of 0.9 cm. This controlled experiment framework allows to adopt a Monte Carlo approach to quantitatively investigate the influence of the wavelength on the performance of the microwave link to retrieve rain rates.

3. VARIABILITY OF THE $< R > - < k >$ RELATION WITH THE WAVELENGTH OF THE LINK

To illustrate the linearity mentioned in the introduction, Figure 2 presents the scatter of $< R > - < k >$ values obtained from the 400 profiles at a 0.9-cm wavelength, for the moderate rain parameterization.

Such a power law can be fitted for every wavelength, and therefore the coefficients of the $< R > - < k >$ relation can be plotted as a function of the wavelength. Figure 3 presents the prefactor and the exponent as a function of the wavelength, for the two types of rainfall. To estimate the variability of this relation, we used a bootstrapping technique: the set of 400 profiles was divided in 20 subsets of 40 profiles (with overlapping), from which the coefficients of the $< R > - < k >$ relation were derived. Then the 10-50-90%

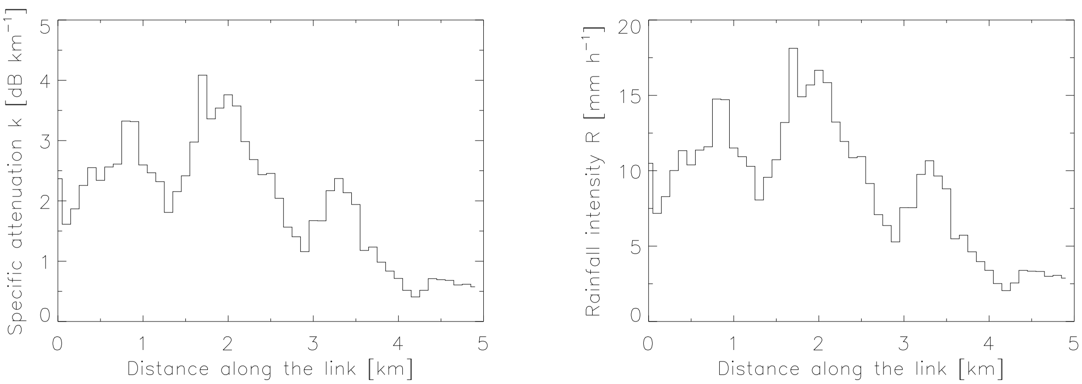

FIGURE 1. Example of generated profile of specific attenuation k at a 0.9-cm wavelength and rain rate R.

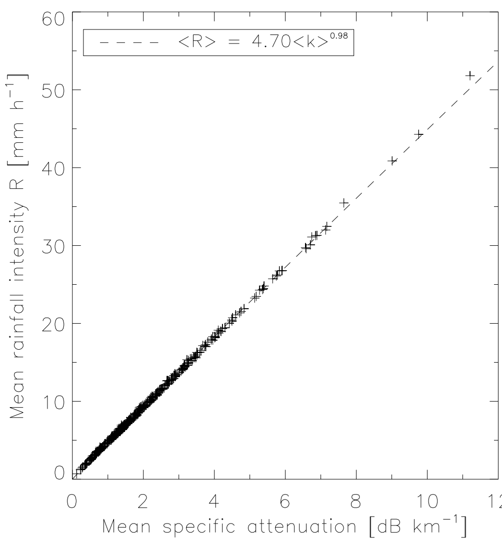

FIGURE 2. Scatter plot of path-average rain rate $< R >$ versus path-average specific attenuation $< k >$. The fitted power law (using a non-linear regression method) is indicated in the plot.

quantiles were estimated. The parameterization of the DSD simulator can be seen to have a limited influence on the wavelength at which the $< R > - < k >$ relation is linear. Therefore, Figure 3 shows that a wavelength of about 0.9 cm appears optimal for rain rate estimation using a microwave link. The prefactor and exponent values for the two DSD parameterizations for 0.9 cm are listed in Table 2. In the following, the wavelength of the link is fixed to 0.9 cm.

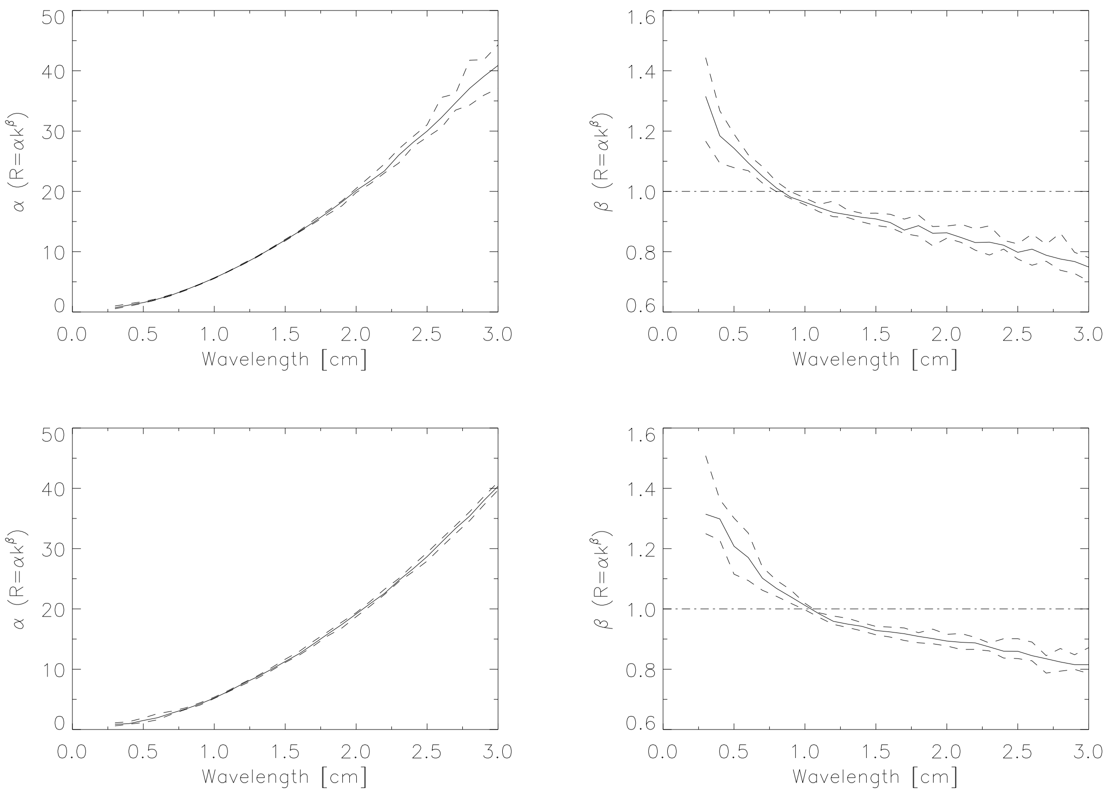

FIGURE 3. Prefactor α (left column) and exponent β (right column) of the relation $< R >= \alpha < k >^{\beta}$ as a function of the wavelength for the moderate rainfall (top panels) and the intense rainfall (bottom panels) parameterization of the DSD simulator. The solid line represents the 50% quantile and the dashed lines represent the 10% and 90% quantiles.

TABLE 2. Median prefactor α and exponent β values at a 0.9 cm wavelength for the two DSD parameterizations (DSD1 corresponds to moderate rainfall, DSD2 corresponds to intense rainfall). The 10% and 90% quantiles are given between brackets.

	DSD1	DSD2
α	4.62 (4.56 ; 4.64)	4.26 (4.05 ; 4.44)
β	0.98 (0.97 ; 1.00)	1.04 (1.02 ; 1.06)

4. UNCERTAINTY ON RAIN RATE RETRIEVAL USING A MICROWAVE LINK

Because we employ a controlled experiment framework, the exact $< R >$ could be used in the previous section. In reality however, the exact $< R >$ is not available, and the problem of estimating the coefficients of the $< R > - < k >$ relation arises. One possible approach to tackle this issue is to deploy a disdrometer somewhere within the link. Such a device will collect time series of point DSD measurements. From a large enough data set (to be representative of the local climatology), it is possible to derive the coefficients of the $R-k$ relation (at the point scale). In order to check how different this 'point scale' relation is with respect to the 'link scale' relation, the distribution of the coefficients of the 'point scale' $R-k$ relation (derived for each profile) is plotted in Figure 4, together with the value corresponding to the coefficients of the 'link scale' $<R>-<k>$ relation. According to Figure 4, the variability of the coefficients of the $R-k$ relation is limited

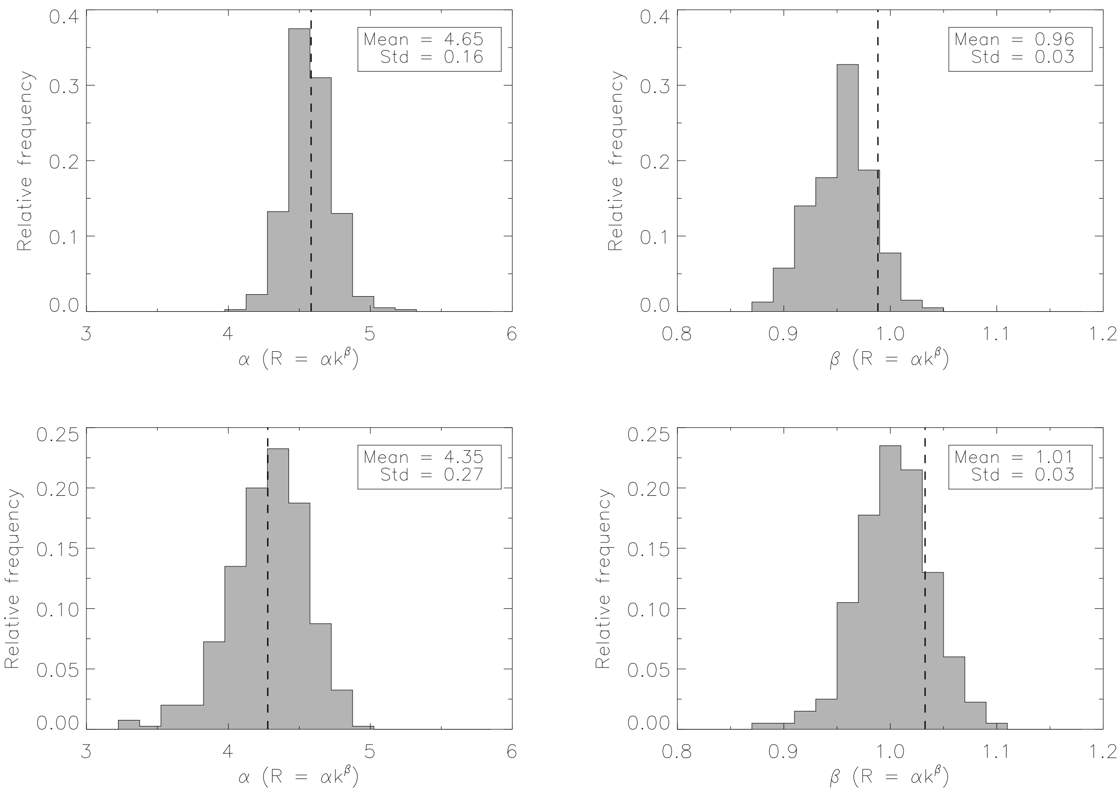

FIGURE 4. Distribution of the values of the prefactor (left panels) and exponent (right panels) of the $R-k$ relation (point scale) over the 400 profiles. The dashed lines show the values of the coefficients of the $<R>-<k>$ relations (link scale). The top (bottom) panels correspond to the moderate (intense) rainfall parameterization of the DSD simulator.

for both types of rainfall. Moreover, the distributions are consistent with the values of prefactor and exponent of the $<R>-<k>$ relations. Therefore, the coefficients of a 'point scale' $R-k$ relation can be used to retrieve the path-average rain rate from the path-average attenuation. It must be noted that the variability of the prefactor (quantified by the standard deviation) is larger for the intense rainfall parameterization.

To quantify the uncertainty associated with this approach, Figure 5 displays the distribution of the relative difference (normalized by the exact $<R>$ value) between the path-average rain rate estimated using the 10% and 90% quantiles of the coefficients of the $R-k$ relation. The mean uncertainty on $<R>$ is about 10% with a standard

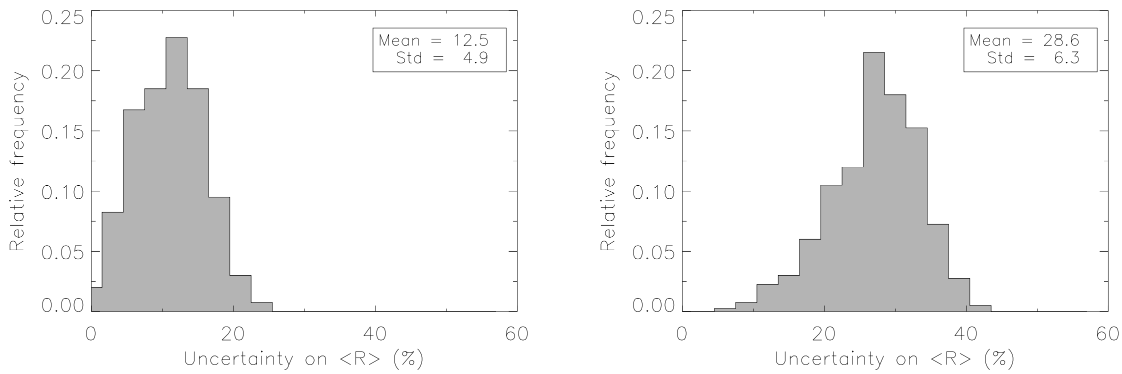

FIGURE 5. Distribution of the uncertainty in $< R >$ retrieval, estimated as the relative difference between the $< R >$ values derived using the 10% and 90% quantiles of the coefficients of the $R - k$ relation, normalized by the exact $< R >$ value. The left (right) panel corresponds to the moderate (intense) rainfall parameterization of the DSD simulator.

deviation of 5% for the moderate rainfall parameterization, but it is about 30% with a standard deviation of 6% for the intense rainfall parameterization of the DSD simulator. This difference is linked to the larger variability in the prefactor for the intense rainfall parameterization (see Figure 4).

5. CONCLUSIONS

In this paper, we investigated the influence of both the wavelength of the microwave link and the spatial variability of the DSD within the path of the link. A stochastic simulator of range profiles of DSD was used to generate 400 profiles at wavelengths ranging from 0.3 to 3 cm, for which the specific attenuation and the rain rate were known. From disdrometric measurements of Mediterranean rainfall, a moderate and an intense rainfall parameterization for the DSD simulator were implemented to study the effect of the type of rainfall.

The retrieval of path-average rain rates using a microwave link is based on the almost linearity of the relation between the path-average specific attenuation $< k >$ and the path-average rain rate $< R >$. For this relation to be linear, the wavelength must be about 0.9 cm for both types of rainfall.

Because the distribution of the coefficients of the $R - k$ relation has a limited dispersion and because it is consistent with the path-average coefficients, we suggest to use a 'local scale' relation derived from disdrometric measurements to convert the attenuation measured by the link into path-average rain rate. The uncertainty associated with this approach, quantified using the quantiles of the distribution of coefficients of the $R - k$ relation, is in the order of 10% for moderate rainfall but can approach about 30% for intense rainfall.

Future developments of this work will concern the comparison with simulated path-average rain rates estimated from rain gauges, and also the parameterization of the DSD simulator for diverse climates.

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References

Beard, K. (1977), Terminal velocity adjustment for cloud and precipitation drops aloft, *J. Atmos. Sci.*, *34*, 1293–1298.

Berne, A., and R. Uijlenhoet (2005), A stochastic model of range profiles of raindrop size distributions: application to radar attenuation correction, *Geophys. Res. Lett.*, *32*, L10803, doi:10.1029/2004GL021899.

Berne, A., G. Delrieu, J.-D. Creutin, and C. Obled (2004), Temporal and spatial resolution of rainfall measurements required for urban hydrology, *J. Hydrol.*, *299*(3-4), 166–179.

Leijnse, H., R. Uijlenhoet, and J. Stricker (2006a), Hydrometeorological application of a microwave link. Part I: evaporation, *Submitted to Water Resour. Res.*

Leijnse, H., R. Uijlenhoet, and J. Stricker (2006b), Hydrometeorological application of a microwave link. Part II: precipitation, *Submitted to Water Resour. Res.*

Uijlenhoet, R., et al. (1999), HYDROMET Integrated Radar Experiment (HIRE): experimental setup and first results, in *29*$^{\text{th}}$ *Int. Conf. On Radar Meteorology, AMS*, pp. 926–930, Montral, Canada.

van de Hulst, H. (1981), *Light scattering by small particles*, 470 pp., Dover.