FUZZY AND OPTIMAL MODELLING OF WATER ENVIRONMENT EVALUATION

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ABSTRACT

The stochastic process, as well as the fuzzy concept and phenomenon, are inevitable in the water environment evaluation. Considering both the randomness and the fuzziness, based on the Principle of Maximum Entropy (POME), and used the concept and method of *Engineering Fuzzy Set Theory*, two weighting generalized distances are defined respectively to build up two fuzzy and optimal models for water environment evaluation. The validity and reliability of model Ⅰ and model Ⅱ are proved by the results of eutrophication evaluation of 12 representative lakes and reservoirs in China. The results of these two models are basically identical, and consistent with the survey outcome. Contrasting the fuzzy model in which only the fuzziness is taken into account, the results of two models constructed here are more detailed, and possess lesser Shannon entropy, which means the smaller uncertainty and more reliability. The theory used and the model constructed in this paper can be extended and applied to other fields.

1. INTRODUCTION

The randomness of characteristic value of water environment evaluation is inevitable during its monitoring, experimenting, and data analyzing, as a result of the physical process, the chemical process and biological process of water contamination are all stochastic process. On the other hand, the fuzziness of water environment evaluation is also inevitable, for the classification standard, the evaluation class and pollution degree is impersonal fuzzy concept and phenomenon.

Entropy is a very important scientific conception. The entropy of a system was first defined by Boltzmann in 1872. Shannon (1948) developed a mathematical theory and applied entropy in the field of communications. Jaynes (1957) formulated the Principle of Maximum Entropy (POME). POME makes good winning in solution the ill-posed problem. Therefore information science has made quite great progress. Now entropy, especially POME has applied in a good many research fields. Paper to review the state-of-the-art in hydrology and water resources has been wrote (Singh, 1997; Wang and Zhu, 2001)

Professor Zadeh (1965)’s Fuzzy Sets has been widely used in many fields (Wang and Han, 1989). In 1998, Professor Chen made a great progress of Zedeh’s theory. He named it Engineering Fuzzy Set Theory (Chen, 1998). This new developed theory provides a new way to ascertain membership degree and membership function. Chen and Xiong (1993) introduced this theory and model of assessment for lake (reservoir) eutrophication.

It is clear that studies with the use of entropy, the Principle of Maximum Entropy (POME) and Engineering Fuzzy Set Theory in hydrology, water resources and water environment have been relatively few. Nevertheless, they are promising and justify further research. It is these studies that provided motivation for our following work.

The objective here is to consider both the randomness and the fuzziness of water environment evaluation, based on the Principle of Maximum Entropy (POME) and the concept and method of Engineering Fuzzy Set Theory. The validity and reliability of model Ⅰ and model Ⅱ constructed this paper are proved by the results of eutrophication evaluation of 12 representative lakes and reservoirs in China.

2. BASIC CONCEPTION AND THEORY

2.1 Informational entropy and the Shannon entropy functional

Informational entropy was first mathematically expressed by Shannon (1948). It has since been called the Shannon Entropy Functional (SEF), denoted as I[f] or I[x]. SEF is a numerical measure of uncertainty associated with the probability density function (pdf) f(x) in describing the random variable x, and defined as

$$I[f] = I[x] = -k\int_a^b f(x)\ln[f(x)/m(x)]dx \tag{1}$$

where k>0 is an arbitrary constant or scale factor depending upon the choice of measurement units, and $m(x)$ is an invariant measure function guaranteeing the invariance of $I[f]$ under any allowable change of variable, and provides an origin measurement of $I[f]$.

The term k can be absorbed into the base of the logarithm and m(x) may be taken as unity so that equation (1) can be as

$$I[f] = -\int_a^b f(x)\ln[f(x)]dx \tag{2}$$

The SEF allows choosing $f(x)$ which minimizes the uncertainty subject to specified constraints. Note that $f(x)$ is conditioned on the constraints used for its derivation.

2.2 the Principle of Maximum Entropy (POME)

The POME formulated by Jaynes states that ‘The minimally prejudiced assignment of probabilities is that which maximizes the entropy subject to the given information.’ Mathematically, it can be stated as follows: Given m linearly independent constraints C_i as

$$C_i = \int_a^b y_i f(x)dx, \qquad i = 1,2,\cdots,m \tag{3}$$

where $y_i(x)$ are some functions whose averages over $f(x)$ are specified.

Then the maximum of subject to the conditions equation (3) is given by

$$f(x) = \exp[-\lambda_0 - \sum_{i=1}^{m}\lambda_i y_i(x)] \tag{4}$$

where $\lambda_i, i = 1,2,\cdots,m,$ are the Lagrange multipliers, and can be determined from equation (3) and equation (4) along with the normalization condition in equation (5).

$$\int_a^b f(x) = 1 \tag{5}$$

2.3 Relative membership degree and function in engineering fuzzy set theory

Since 1965, when Professor Zedeh wrote the first thesis, Fuzzy Sets, great achievements have been made in this new science branch. Now this theory is widely used in many scientific and technology fields, such as in probability, statistics, pattern recognition, clustering analysis, etc. In 1998, Professor Chen made a great progress of Zedeh's fuzzy set theory. He named it Engineering Fuzzy Set Theory (Chen, 1998). This new developed theory provides us a new way to ascertain membership degree and membership function, which blocked up the further development of fuzzy set theory in the past decades. And we employed the concept and method of Chen's theory in this study.

If there is a fuzzy subset or fuzzy concept $\underset{\sim}{A}$ in discussing domain, let the two apices of $\underset{\sim}{A}$ take 0 and 1 to form a continuum on the closed interval [0, 1]. Then establish a reference system on the number axis of this continuum, let a couple of arbitrary points of the reference system as the two apices of its coordinate and take 0 and 1, so that a reference continuum on the number axis [0,1] of reference system.

$\forall\ u \in U$, a number $\mu_{\underset{\sim}{A}}(u)$ is designated in the reference continuum, and named as the relative membership degree of u to $\underset{\sim}{A}$, and the following mapping is named as the relative membership function of $\underset{\sim}{A}$

$$\left.\begin{array}{l} \mu_{\underset{\sim}{A}} : U \rightarrow [0, \quad 1] \\ \quad u \mapsto \mu_{\underset{\sim}{A}}(u) \in [0, \quad 1] \end{array}\right\} \tag{6}$$

3. FUZZY AND OPTIMAL MODELLING OF WATER ENVIRONMENT EVALUATION

3.1 Prophase data disposing

If the number of water environment evaluation classification standard is c, the number of water environment evaluation index is m, the concentration value of each index of classification standard is yih, then there is concentration value matrix of water environment evaluation classification standard Y: $[y_{ih}]_{m\times c}$.

Now there are n water samples to be evaluated, in each sample there are monitoring value xij of m index, then the monitoring concentration value matrix of these samples X: $[x_{ij}]_{m\times n}$ is gotten. If the following rule is regulated, that in class 1, the relative membership degree $s_{i1} = 0$, which is of the standard concentration yi1 of index i towards some evaluation conception $\underset{\sim}{A}$, and in class c, the relative membership degree $s_{ic} = 1$, which is of the standard concentration yic of index i towards this evaluation conception $\underset{\sim}{A}$, then by equation (7), the relative membership degree sih is denoted, which is of the standard concentration yih of index i in class h.

$$s_{ih} = \frac{y_{ih} - y_{i1}}{y_{ic} - y_{i1}} \tag{7}$$

Thereby the concentration value matrix of water environment evaluation classification standard Y is transformed to the relative membership degree concentration value matrix of classification standard $\underset{\sim}{\mathbf{S}}$: $[s_{ih}]_{m\times c}$.

The monitoring value x_{ij} can be transformed to the corresponding relative membership degree r_{ij} with equation (8) or (9)

$$r_{ij}=\begin{cases}1 & x_{ij}>y_{ic}\\ \dfrac{x_{ij}-y_{i1}}{y_{ic}-y_{i1}} & y_{i1}\le x_{ij}\le y_{ic}\\ 0 & x_{ij}<y_{i1}\end{cases} \tag{8}$$

$$r_{ij}=\begin{cases}1 & x_{ij}<y_{ic}\\ \dfrac{x_{ij}-y_{i1}}{y_{ic}-y_{i1}} & y_{i1}\ge x_{ij}\ge y_{ic}\\ 0 & x_{ij}>y_{i1}\end{cases} \tag{9}$$

Consequently, the monitoring value matrix of samples to be evaluated X: $[x_{ij}]_{m\times n}$, is transformed to the relative membership degree monitoring value matrix $\underset{\sim}{\mathbf{R}}$: $[r_{ij}]_{m\times n}$.
Furthermore, the influence weight of each index is denoted by index weight vector $\vec{v}$

$$\vec{v}=(v_1,v_2,\cdots,v_m),\ \sum_{i=1}^{m}v_i=1 \tag{10}$$

Then the synthesis weight matrix A: $[v_i v_{ij}]_{m\times n}$ can be constructed. And the index synthesis weight matrix W: $[w_{ij}]_{m\times n}$ also can be constructed, when A return-to-1 according to its column. If the relative membership degree matrix of n samples towards class c is $\underset{\sim}{\mathbf{U}}$: $[u_{hj}]_{c\times n}$, then its condition of constraint is as follows

$$\sum_{h=1}^{c}u_{hj}=1 \quad u_{hj}\ge 0,\quad j=1,2,\cdots,\ n \tag{11}$$

The number of such matrix is incalculable. And the intention of water environment evaluation is to make the fuzzy optimal matrix $\underset{\sim}{\boldsymbol{U}}$ be consistent with equation (11).

3.2 Fuzzy optimal evaluation model Ⅰ

The stochastic process, as well as the fuzzy concept and phenomenon, are inevitable in the water environment evaluation. So the confirmation of $[u_{hj}]_{c\times n}$ also has such uncertainties as the randomness and the fuzziness. Hereinto, the randomness includes at least two significations. One is that the randomness during evaluation using the concept of relative membership degree. The other is that the randomness of monitoring value error during observation. As the latter is concerned, it is discussed by the use of Monte Carlo method to do some more study in another paper (Wang and Zhu, 2003). Here, randomness during evaluation using the concept of relative membership degree is taken into account.

For the meaning of u_{hj} is that the relative membership degree of sample *j* belong to class *h*, and if we treat u_{hj} as the probability of sample *j* belonging to class *h*, then Shannon entropy can be employed to describe and compare this stochastic uncertainty

$$H_j=-\sum_{h=1}^{c}u_{hj}\ln u_{hj} \tag{12}$$

Definition The difference of sample *j* to class *h* is expressed as weighting generalized distance ${}^{1}D_{hj}$ in the form

$$^{1}D_{hj}=u_{hj}\,{}^{1}d_{hj}=u_{hj}\{\sum_{i=1}^{m}[w_{ij}(r_{ij}-s_{ih})]^{p}\}^{\frac{1}{p}} \tag{13}$$

The confirmation of $[u_{hj}]_{c\times n}$ should also minimize the sum of weighting generalized distance of whole samples to each class of classification standard, namely

$$\begin{aligned}&\min_{u_{hj}} \quad {}^{1}D=\sum_{j=1}^{n}\sum_{h=1}^{c}u_{hj}\{\sum_{i=1}^{m}[w_{ij}(r_{ij}-s_{ih})]^{p}\}^{\frac{1}{p}}\\ &s.t. \quad \sum_{h=1}^{c}u_{hj}=1\\ &\qquad u_{hj}\geq 0, \quad j=1,2,\cdots, \quad n\end{aligned} \tag{14}$$

At the same time, based on the Principle of Maximum Entropy (POME), the confirmation of $[u_{hj}]_{c\times n}$ should also maximize Shannon entropy

$$\begin{aligned}&\max_{u_{hj}} \quad H=\sum_{j=1}^{n}(-\sum_{h=1}^{c}u_{hj}\ln u_{hj})\\ &s.t. \quad \sum_{h=1}^{c}u_{hj}=1\\ &\qquad u_{hj}\geq 0, \quad j=1,2,\cdots, \quad n\end{aligned} \tag{15}$$

This is a dual-objective programming problem. To solve it, we may use weighting method, constraint method, hybrid method, etc (Compiling Committee of Modern Applied Math Notebook, 1998). In this paper, weighting method is used. So a single-objective programming is constructed as follows, where η_1 is the weighting factor.

$$\begin{aligned}&\min_{u_{hj}} \quad Y={}^{1}D+\frac{1}{\eta_1}H=\sum_{j=1}^{n}\sum_{h=1}^{c}\left\{u_{hj}\{\sum_{i=1}^{m}[w_{ij}(r_{ij}-s_{ih})]^{p}\}^{\frac{1}{p}}+\frac{1}{\eta_1}u_{hj}\ln u_{hj}\right\}\\ &s.t. \quad \begin{cases}\sum_{h=1}^{c}u_{hj}=1\\ u_{hj}\geq 0, \quad j=1,2,\cdots, \quad n\end{cases}\end{aligned} \tag{16}$$

The optimal solution should be consistent with Kuhn-Tucker conditions. Changing the value of η_1, and solving equation (16) time after time, then noninferior solution set of the dual-objective programming problem can be gotten. The Lagrange function of this programming problem is as follows, Where λ_1 is the Lagrange multiplier.

$$L(u_{hj},\lambda_1)=\sum_{j=1}^{n}\sum_{h=1}^{c}\left\{u_{hj}\{\sum_{i=1}^{m}[w_{ij}(r_{ij}-s_{ih})]^{p}\}^{\frac{1}{p}}+\frac{1}{\eta_1}u_{hj}\ln u_{hj}\right\}+\lambda_1\left(\sum_{h=1}^{c}u_{hj}-1\right) \tag{17}$$

Differentiating equation (17) with respect to u_{hj} and vanish as

$$\frac{\partial L}{\partial u_{hj}}=\{\sum_{i=1}^{m}[w_{ij}(r_{ij}-s_{ih})]^{p}\}^{\frac{1}{p}}+\frac{1}{\eta_1}(\ln u_{hj}+1)+\lambda_1=0 \tag{18}$$

Resulting in

$$u_{hj}=\exp[-\eta_1\{\sum_{i=1}^{m}[w_{ij}(r_{ij}-s_{ih})]^{p}\}^{\frac{1}{p}}-\eta_1\lambda_1-1] \tag{19}$$

Differentiating equation (17) with respect to λ_1 and vanish as

$$\frac{\partial L}{\partial \lambda_1} = \sum_{h=1}^{c} u_{hj} - 1 = 0 \tag{20}$$

From equation (19) and equation (20)

$$\exp[-(\eta_1 \lambda_1 + 1)] = \left(\sum_{h=1}^{c} \exp[-\eta_1 \{\sum_{i=1}^{m} [w_{ij}(r_{ij} - s_{ih})]^p\}^{\frac{1}{p}}] \right)^{-1} \tag{21}$$

By inserting equation (21) in equation (19)

$$u_{hj} = \exp[-\eta_1 \{\sum_{i=1}^{m} [w_{ij}(r_{ij} - s_{ih})]^p\}^{\frac{1}{p}}] \left(\sum_{h=1}^{c} \exp[-\eta_1 \{\sum_{i=1}^{m} [w_{ij}(r_{ij} - s_{ih})]^p\}^{\frac{1}{p}}] \right)^{-1} \tag{22}$$

3.3 Fuzzy optimal evaluation model Ⅱ

Definition The difference of sample *j* to class *h* is expressed as weighting generalized distance ${}^2D_{hj}$ in the form

$${}^2D_{hj} = u_{hj}\,{}^2d_{hj} = u_{hj}[\sum_{i=1}^{m} (w_{ij}|r_{ij} - s_{ih}|)] \tag{23}$$

In the same way, a single-objective programming is constructed as follows

$$\min_{u_{hj}} \quad Y = {}^2D + \frac{1}{\eta_2} H = \sum_{j=1}^{n} \sum_{k=1}^{c} \left\{ u_{hj} [\sum_{i=1}^{m} (w_{ij}|r_{ij} - s_{ih}|)] + \frac{1}{\eta_2} u_{hj} \ln u_{hj} \right\}$$

$$s.t. \quad \begin{cases} \sum_{h=1}^{c} u_{hj} = 1 \\ u_{hj} \geq 0, \quad j = 1, 2, \cdots, \ n \end{cases} \tag{24}$$

Where η_2 is the weighting factor.

The Lagrange function of this programming problem is in the form

$$L(u_{hj}, \lambda_2) = \sum_{j=1}^{n} \sum_{h=1}^{k} \left\{ u_{hj} [\sum_{i=1}^{m} (w_{ij}|r_{ij} - s_{ih}|)] + \frac{1}{\eta_2} u_{hj} \ln u_{hj} \right\} + \lambda_2 \left(\sum_{h=1}^{k} u_{hj} - 1 \right) \tag{25}$$

Where λ_2 is the Lagrange multiplier.

Finally,

$$u_{hj} = \exp[-\eta_2 \sum_{i=1}^{m} (w_{ij}|r_{ij} - s_{ih}|)] \left(\sum_{h=1}^{c} \exp[-\eta_2 \sum_{i=1}^{m} (w_{ij}|r_{ij} - s_{ih}|)] \right)^{-1} \tag{26}$$

4. CASES

In 1990, Shu did a study on the water eutrophication evaluation. Using his data and the equations above, taking *p=2*, the value of the fuzzy optimal evaluation model Ⅰ and Ⅱ is as equation (27) and (28) respectively.

The results of eutrophication evaluation of 12 representative lakes and reservoirs in China are listed in Table 1. The evaluation results of model Ⅰ and model Ⅱ constructed are basic identical, except evaluation results of sample 3 have a little different, and consistent with the survey outcome. Contrasting the fuzzy model in which only the fuzziness is taken into account, the results of model Ⅰ and model Ⅱ constructed this paper are more detailed. For examples, as sample 2, sample 4 and sample 6 are concerned, the results of model Ⅰ and Ⅱ give us more information about the eutrophication status.

$$\underset{\sim}{\mathbf{U}}_1^T = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \\ 8 \\ 9 \\ 10 \\ 11 \\ 12 \end{array} \begin{array}{c} \begin{array}{cccccc} \textit{little} & \textit{a little} & \textit{mid} & \textit{much} & \textit{more} & \textit{most} \end{array} \\ \begin{bmatrix} 0 & 0.1470 & 0.5918 & 0.2118 & 0.0487 & 0.0007 \\ 0 & 0.0050 & 0.7875 & 0.1815 & 0.0239 & 0.0021 \\ 0 & 0.0003 & 0.4777 & 0.4499 & 0.0717 & 0.0004 \\ 0 & 0.0001 & 0.3403 & 0.5928 & 0.0664 & 0.0003 \\ 0 & 0.0002 & 0.1866 & 0.4860 & 0.3264 & 0.0007 \\ 0 & 0.0003 & 0.0710 & 0.1610 & 0.7668 & 0.0009 \\ 0 & 0.0006 & 0.0132 & 0.0258 & 0.9595 & 0.0008 \\ 0 & 0.0014 & 0.0078 & 0.0176 & 0.9613 & 0.0118 \\ 0 & 0.0007 & 0.0065 & 0.0133 & 0.9775 & 0.0019 \\ 0 & 0 & 0 & 0 & 0.0002 & 0.9998 \\ 0 & 0 & 0 & 0 & 0.0012 & 0.9988 \\ 0 & 0 & 0 & 0 & 0.0003 & 0.9997 \end{bmatrix} \end{array} \tag{27}$$

$$\underset{\sim}{\mathbf{U}}_2^T = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \\ 8 \\ 9 \\ 10 \\ 11 \\ 12 \end{array} \begin{array}{c} \begin{array}{cccccc} \textit{little} & \textit{a little} & \textit{mid} & \textit{much} & \textit{more} & \textit{most} \end{array} \\ \begin{bmatrix} 0 & 0.1671 & 0.4902 & 0.2770 & 0.0655 & 0.0002 \\ 0 & 0.0274 & 0.6622 & 0.2875 & 0.0225 & 0.0004 \\ 0 & 0.0028 & 0.4936 & 0.4552 & 0.0484 & 0 \\ 0 & 0.0020 & 0.3234 & 0.6315 & 0.0430 & 0.0001 \\ 0 & 0.0017 & 0.1849 & 0.5747 & 0.2385 & 0.0002 \\ 0 & 0.0011 & 0.0752 & 0.2250 & 0.6982 & 0.0005 \\ 0 & 0.0004 & 0.0105 & 0.0321 & 0.9576 & 0.0003 \\ 0 & 0.0002 & 0.0024 & 0.0076 & 0.9849 & 0.0049 \\ 0 & 0.0002 & 0.0038 & 0.0116 & 0.9893 & 0.0005 \\ 0 & 0 & 0 & 0 & 0.0001 & 0.9999 \\ 0 & 0 & 0 & 0 & 0.0005 & 0.9995 \\ 0 & 0 & 0 & 0 & 0.0001 & 0.9999 \end{bmatrix} \end{array} \tag{28}$$

TABLE 1 Results of eutrophication evaluation of representative lakes(reservoirs) in China

No.	Name	Model Ⅰ	Model Ⅱ	fuzzy model
1	Qinghai	Mid	Mid	Mid
2	Erhai	Mid(partial Much)	Mid(partial Much)	Mid
3	Bositeng lake	Mid(partial Much)	Much(partial mid)	Mid(partial Much)
4	Yuqiao reservoir	Much(partial mid）	Much(partial mid）	Much
5	Cihu lake	Much(partial more)	Much(partial more)	Much(partial more)
6	Chaohu lake	More(partial Much)	More(partial Much)	More
7	Gantang lake	More	More	More
8	Mogu lake	More	More	More
9	West lake	More	More	More
10	Xuanwu lake	Most	Most	Most
11	Moshui lake	Most	Most	Most
12	Dongshan lake	Most	Most	Most

Shannon entropy of each model is listed in Table 2. The Shannon entropy of model Ⅰ and model Ⅱ constructed this paper are lesser than the model in which only the fuzziness is taken into account, except sample 4, sample 5 and sample 6, which means the smaller uncertainty and more reliability of model Ⅰ and model Ⅱ.

TABLE 2 Shannon entropy of each evaluation model

No.	Name	Shannon entropy of model Ⅰ	Shannon entropy of model Ⅱ	Shannon entropy of fuzzy model
1	Qinghai	1.0730	1.1852	1.3985
2	Erhai	0.6261	0.8186	0.8292
3	Bositeng lake	0.9071	0.8706	0.9128
4	Yuqiao reservoir	0.8607	0.8040	0.6508
5	Cihu lake	1.0361	0.9851	0.6631
6	Chaohu lake	0.6942	0.7921	0.5798
7	Gantang lake	0.2017	0.2030	0.2964
8	Mogu lake	0. 2091	0.0943	0.5683
9	West lake	0.1300	0.0944	0.4186
10	Xuanwu lake	0.0018	0.0008	0.2542
11	Moshui lake	0.0094	0.0043	0.4804
12	Dongshan lake	0.0030	0.0008	0.1742

5. EPILOGUE

In this paper, considering both the randomness and the fuzziness, based on the Principle of Maximum Entropy (POME), and used the concept and method of *Engineering Fuzzy Set Theory*, two weighting generalized distances are defined respectively to build up two fuzzy and optimal models for water environment evaluation. The validity and reliability of model Ⅰ and model Ⅱ constructed this paper are proved by the results of eutrophication evaluation of 12 representative lakes and reservoirs in China. The theory used and the model constructed in this paper can be extended and applied to other fields.

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