INVERSE MODELING OF ELECTRICAL CONDUCTIVITY DISTRIBUTIONS FROM ERT DATASETS: INTEGRATED ANALYSES OF THE SUCCESSIVE LINEAR ESTIMATOR AND A SMOOTHNESS CONSTRAINED REGULARIZATION APPROACH

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Abstract

Electrical resistivity tomography data sets can be converted to spatial distributions of electrical resistivities using different inversion procedures. We tested the quality of two different inversion approaches: a regularization approach and a geostatistical based approach, the successive linear estimator (SLE). Furthermore, the SLE was used to investigate effects of conditioning in the inversion. The results of our investigations showed that the quality of an inversion is as expected, strongly depending on the measurement geometry and the number of measurements. An integrated analyses of the inverse modeling results suggest that, with only sparse measurements the conditioning of the electrical inverse problem with point values of the plume concentration, enhances the quality of the result clearly. However, with an increasing number of measurements the effect of conditioning becomes less effective.

1. INTRODUCTION

Recent field and synthetic studies showed that electrical resistivity tomography (ERT) potentially is a viable tool for characterizing subsurface transport processes of substances with electrical conductivities differing from the background [2, 8]. Popular approaches for interpreting the electrical field measurements are based on inversion procedures including regularization terms, which force the inverse solution to be smooth. As a result, the image of a solute plume can be unsatisfactory. Furthermore most inversion procedures do not generally take advantage of in situ electrical conductivity measurements or point concentration data. We further recognize that while a solute plume in heterogeneous aquifers can be highly irregular, it can be characterized in a geostatistical sense: its mean position, lateral spreading, and spatial correlation structures. These plume statistics can serve as our prior knowledge about the plume. Based on existing stochastic theories of solute transport processes, plume statistics can be easily estimated from statistics, quantifying the heterogeneity of the hydraulic conductivity [11, 5]. Therefore, we hypothesize that an ERT inversion, incorporating our prior knowledge of the geostatistical characteristics of a plume and some direct point measurements of the plume concentration, could lead to more detailed images of subsurface electrical conductivity distributions associated with the solute plume.

2. ERT INVERSION TECHNIQUES

To survey the distribution of electrical conductivity in the subsurface by ERT, voltages induced by currents are measured. Subsequently these measurements are used in inversion procedures to estimate the electrical conductivity in the subsurface. In geophysical sciences a broad variety of inversion techniques are known. For ERT, the inversion techniques are commonly based on regularization seeking the "smoothest" model fitting the measured data. More recently geostatistical approaches for inversion are investigated. In the present study we used a smoothness constrained regularization based [1, 3, 2] and the SLE, a geostatistically based approach [10]. Both are proceeding towards the final estimate of the electrical conductivity distribution by looping between forward modeling, comparison with measurement data and updating the estimate of the electrical resistivity distribution. To highlight the major differences between the two approaches the updating process is shown for both. For a more detailed description of the inversion approaches we refer to the afore cited literature on the used inversion approaches. Applying the regularization based inversion, the updated electrical conductivity distribution $\boldsymbol{\chi}^{\boldsymbol{r+1}}$ is computed by

$$\boldsymbol{\chi}^{r+1} = \boldsymbol{\chi}^{r} + [\boldsymbol{J}^{T}\boldsymbol{J} + \alpha \boldsymbol{R}^{T}\boldsymbol{R}]^{-1}[\boldsymbol{J}^{T}(\boldsymbol{v}^{*} - \boldsymbol{v}^{r}) - \alpha \boldsymbol{R}^{T}\boldsymbol{R}\boldsymbol{\chi}^{r}], \tag{1}$$

depending on the prior estimate of the electrical conductivity distribution $\boldsymbol{\chi}^{\boldsymbol{r}}$, the sensitivity- (Jacobian-) matrix $\boldsymbol{J}$, a regularization parameter α, a matrix evaluating the roughness of $\boldsymbol{\chi}^{\boldsymbol{r}}$ denoted as $\boldsymbol{R}$ and the difference between estimated and measured voltages $\boldsymbol{v}^{*} - \boldsymbol{v}^{\boldsymbol{r}}$. Using the SLE, the updated electrical conductivity distribution $\boldsymbol{\chi}^{\boldsymbol{r+1}}$ is computed by

$$\boldsymbol{\chi}^{r+1} = \boldsymbol{\chi}^{r} + \boldsymbol{\epsilon}^{r}_{\Delta\chi\Delta\chi}\boldsymbol{J}^{T}[\boldsymbol{J}\boldsymbol{\epsilon}^{r}_{\Delta\chi\Delta\chi}\boldsymbol{J}^{T}]^{-1}[\boldsymbol{v}^{*} - \boldsymbol{v}^{r}], \tag{2}$$

depending on the prior estimate of the electrical conductivity distribution $\boldsymbol{\chi}^{\boldsymbol{r}}$, the conditional covariance of the residuals of the electrical conductivity $\boldsymbol{\epsilon}^{\boldsymbol{r}}_{\boldsymbol{\Delta\chi\Delta\chi}}$, the Sensitivity- (Jacobian-) matrix $\boldsymbol{J}$ and the difference between estimated and measured voltages $\boldsymbol{v}^{*} - \boldsymbol{v}^{\boldsymbol{r}}$.

The updating process within the inversion procedures shows the different strategies to handle the apparent problem of illposedness or the non uniqueness of the inverse problem. The regularization approach uses a smoothness constrain in addition to the minimization of the difference $\boldsymbol{v}^{*} - \boldsymbol{v}^{\boldsymbol{r}}$. By contrast the SLE utilizes the conditional covariance of the residuals the electrical conductivity together with the minimization of the difference $\boldsymbol{v}^{*} - \boldsymbol{v}^{\boldsymbol{r}}$.

3. SYNTHETIC EXPERIMENTS

Based on a synthetically generated two dimensional heterogeneous hydraulic conductivity field we first modeled a flow field using the finite element code TRACE [9]. In a second step we simulated non-reactive solute transport within this flow field using the particle tracking code PARTRACE [6, 4]. In a third step we extracted the concentration distribution for a given time of the transport simulation. In a last step the concentrations are transformed to electrical conductivities using a linear relationship. These procedures led to an electrical conductivity distribution with a background electrical conductivity of

0.01 S/m and a maximum electrical conductivity within the plume of 0.1 m/S as shown in Figure 1 a). In the following this electrical conductivity distribution is denoted as original.

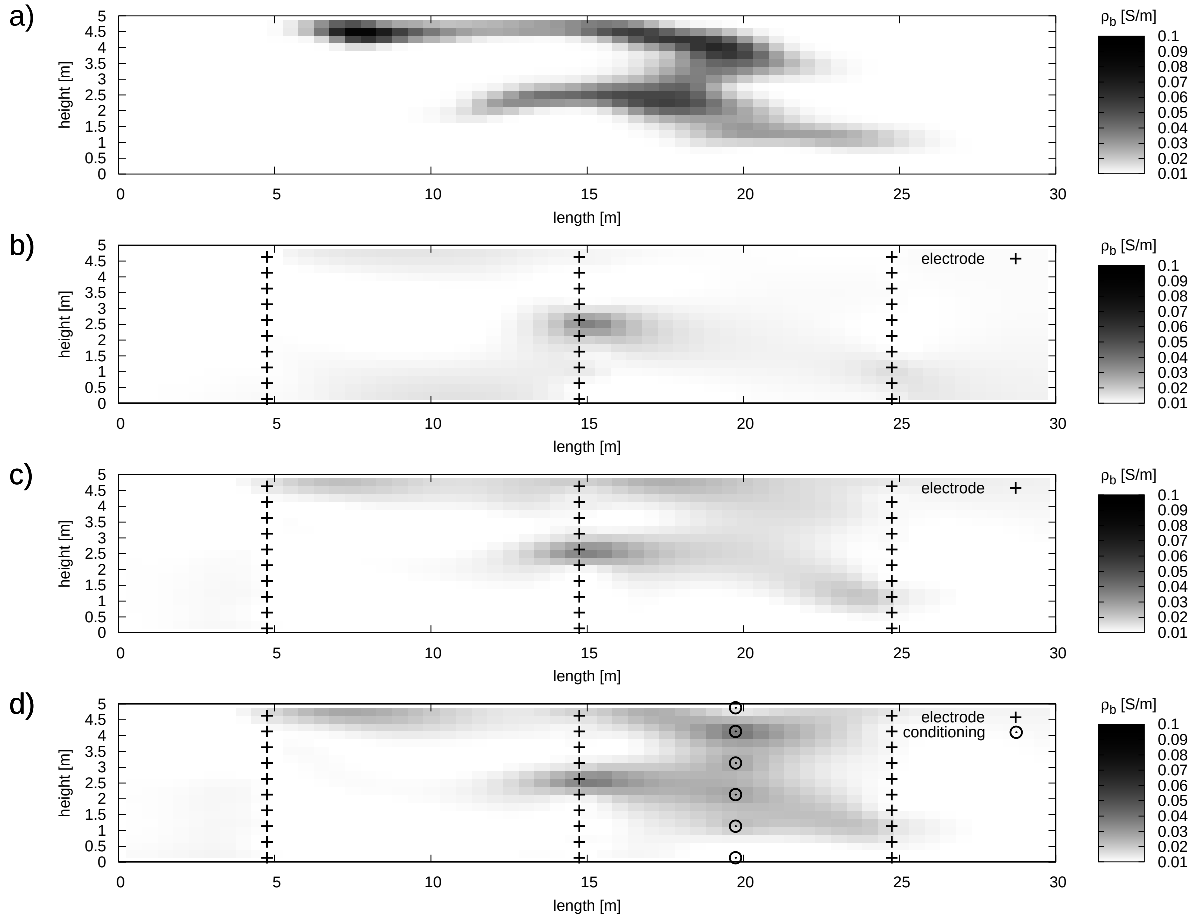

FIGURE 1. Two dimensional electrical conductivity distributions: a) synthetically simulated tracer plume, b) inversion results using the regularization technique without conditioning c) inversion results using the SLE technique without conditioning, d) inversion results using the SLE technique including conditioning. The electrode positions are denoted by crosses. The locations of a priori known electrical conductivities are denoted by circles.

The generation of the original synthetic electrical conductivity field was succeeded by implementing different electrode arrays within the field. In Figure 1 b) c) d) the electrode array for the three borehole case is shown, being equipped with three electrode chains. The distance between the electrodes within one electrode chain was set to 0.5 m. In the five borehole case, two electrode chains are additionally installed at 10 m and 20 m (not shown in Figure 1). The nine borehole case is additionally equipped with electrode chains at the positions 7.5 m, 12.5 m, 17.5 m and 22.5 m (not shown in Figure 1).

Based on the original electrical conductivity field, we performed ERT forward modeling or in other words a synthetic ERT measurement for each particular case. Thereto we used a so called “skip one” dipole-dipole scheme [7]. This means that all electrode pairs used as dipoles are 1 m distant in vertical direction. In the scope of this scheme the “skip one” dipole current was moved in 0.5 m steps along each electrode chain. In the borehole of the current and the neighboring boreholes, voltages are measured simultaneously at all “skip one” dipoles at least one meter distant from the current. This resulted in 24 currents and 316 synthetic voltage measurements for the three borehole case. The total number was 40 and 612 in the five borehole case and 72 and 1204 in the nine borehole case for currents and voltage measurements, respectively. Furthermore we assumed that at six positions the electrical conductivity is known a priori or in other words we synthetically measured the electrical conductivity (see Figure 1 d)).

In order to estimate electrical conductivity fields based on the synthetic ERT surveys we performed inversions using both the regularization approach and the SLE. In Figure 1 b) c) d) the inversion results are shown for the three borehole case. Due to the sparse number of electrodes the estimated electrical conductivity fields are quite weak. However, the three borehole case shows clearly that the SLE can improve inversion results and moreover the usage of a priory knowledge of electrical conductivities at some points enhances the inversion results. Quantitatively this is expressed in an increase of the correlation coefficient of log transformed electrical conductivities from the original and the estimated fields (see Table 1).

Table 1. Correlation Coefficient Between Inversion Results and the Original Electrical Conductivity Distribution

Number of Boreholes	Regularization	SLE without conditioning	SLE conditioned
3	0.38	0.54	0.70
5	0.75	0.85	0.85
9	0.89	0.92	0.92

As expected, the correlation coefficients presented in Table 1 obviously show that the increase of the number of electrodes used in the ERT survey enhances the result. Furthermore, the correlation coefficients in Table 1 indicate that with increasing number of electrodes the beneficial effects of conditioning become less pronounced.

4. CONCLUSIONS AND OUTLOOK

Although the presented synthetic experiments are two dimensional and therefore can be interpreted in a qualitative sense only, the results suggest that the SLE is promising for ERT inversion. Moreover the utilization of a priori knowledge in the inversion procedure can improve inversion results. However, further work is needed to adopt the presented findings to real field ERT data: Detailed analyses of the estimated and the original plume statistics will provide a more transport related quantification of the quality of inversion results. Due to the fact that field measurements are afflicted with errors, integrated analyses of the inverse procedures using noisy input data will provide a more realistic analyses of the quality of the inverse approaches. Finally, because flow, transport and electrical processes are three dimensional, three dimensional field and synthetic studies will supply quantitative findings regarding the quality of inverse approaches.

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