AN INTEGRATED MEDIA, INTEGRATED PROCESSES WATERSHED MODEL – WASH123D: PART 3 –A COMPARATIVE STUDY ON DIFFERENT SURFACE WATER/GROUNDWATER COUPLING APPROACHES

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ABSTRACT

In the core of an integrated watershed model is the coupling among surface water and subsurface water flows. Recently, there is a tendency of claiming the fully coupled approach for surface water and groundwater interactions in the hydrology literature. One example is the assumption of a gradient type flux equation based on Darcy's law (linkage term) and the numerical solution of all governing equations in a single global matrix. We argue that this is only a special case of all possible coupling combinations and if not applied with caution, the non-physics interface parameter becomes a calibration tool. Generally, there are two cases based on physical nature of the interface: continuous or discontinuous assumption, when a sediment layer exists at the interface, the discontinuous assumption may be justified. As for numerical schemes, there are three cases: time-lagged, iterative, and simultaneous solutions. Since modellers often resort to the simplest, fastest schemes in practical applications, it is desirable to quantify the potential error and performance of different coupling schemes. We evaluate these coupling schemes in a finite element watershed model, WASH123D. Numerical experiments are used to compare the performance of each coupling approach for different types of surface water and groundwater interactions. These are in term of surface water and subsurface water solutions and exchange flux (e.g. infiltration/seepage rate). It is concluded that different coupling approaches are justified for flow problems of different spatial and temporal scales and the physical setting of the interface.

1. INTRODUCTION

In the core of an integrated, physics-based watershed model is the coupling among surface water and subsurface water flows. In this study, we focus on the coupling approaches for such kind of physics-based models that partial differential equations are applied for both surface water flows (Saint Venant equations) and variably saturated subsurface flow (Richards equation). For simpler watershed models, the infiltration term is treated by an empirical algebraic equation; accordingly the surface water/groundwater interaction computation is straightforward and independent of water flow solution. Although the numerical solution of the Richards equation is computationally intensive and difficult at watershed scale, it is essential for a complete, physics-based representation of the subsurface flow.

The consideration of surface water/subsurface water interactions consists of two parts: stream-aquifer interaction and overland-subsurface interaction. The stream-aquifer interaction has been extensively studied by coupling saturated groundwater flow with 1-D channel flow (for example, Pinder and Sauer, 1971; Swain and Wexler, 1996, among others). The role of unsaturated zone is rarely considered. The overland-subsurface interaction is usually studied in the context of runoff generation at the watershed scale (for example, Freeze, 1972; Smith and Woolhiser, 1971; Akan and Yen, 1981 and Abott et al., 1986).

Recently, the physics-based coupling of surface and subsurface flows has been an active research topic. Panday and Huyakorn (2004) discussed the utility of different coupling schemes in a new watershed model. Yen and Morita (2002) classified coupling approaches as fully coupled (simultaneous solution), alternating iterative coupling and externally coupling (decoupled). In their model, the concept of infiltrability was introduced; however, a leakage term was used in the coupling process.

With the assumption of a linkage term at the interface, the full implicit coupling approach has been tested in some recent models: InHM (VanderKwaak, 1999), MODHMS (Panday and Huyakorn, 2004), and Aral and Gaunduz (2005). Enhanced numerical stability and accuracy were reported on some test examples as a result of solving the coupled governing equations in a single global matrix. However, this approach also has intrinsic limitations. An assumption has to be made that there exists a weak permeable layer at the interface and identical time steps must be used for both surface water and subsurface flow.

Kollet and Maxwell (2005) pointed out the limitation of the linakge term approach and considered overland flow as part of subsurface flow boundary condition. However, in their coupled model, the same time step was used for both overland and subsurface flow.

The time-lagged, decoupled approach is still very popular in some loosely coupled models that consist of an existing surface water model and a subsurface flow counterpart. Earlier studies make use of this approach to reduce computation effort. Smith and Woolhiser (1971) coupled one-dimensional Richards equation with one-dimensional kinematic wave overland flow. Singh and Bhallamudi (1997) developed a coupled overland flow model with 1-D dynamic wave equations and 2-D Richards equation. The time-lagged, decoupled approach was used.

Although all the coupling approaches are conceptually simple, there seems to be no consensus on what is the best approach in the hydrology literature. The so-called fully coupled approach has different definition and meaning for different people in current hydrologic modeling research and practice.

It is also interesting to see how different coupling approaches can reach similar results for the example problem of Smith and Woolhiser (1971). In their original paper, kinematic wave approximation was applied for overland flow and a time-lagged, decoupled approach was used. Singh and Bhallamudi (1997) applied the full St. Venant equations for overland flow with a decoupled scheme. Morita and Yen (2001) applied the iterative coupling approach and VanderKwaak (1999) applied a linkage-term based simultaneous approach. All models reported close match with the experiment data. A possible explanation is that the example is a typical Hortonian runoff problem and subsurface flow is not important, so the surface/subsurface interaction is weak.

2. GOVERNING EQUATIONS AND INTERFACE CONDITIONS

2. 1 Overland and channel flows.

The diffusion wave approximation of the Saint Venant equations for surface water flows can be stated as:

$$B\frac{\partial H}{\partial t} - \nabla \cdot [\mathbf{K}(\nabla H)] = S + R - E - I \tag{1}$$

where B is the top width of the channel cross-section for 1-D channel flow and B=1 for overland flow. H is water level and K a diffusion coefficient that is a function of water depth; S is human induced source/sink, R is rainfall, E is evapotranspiration and I is the infiltration term.

2.2 Three-dimensional subsurface flow.

The governing equation of subsurface flow through saturated-unsaturated porous media can be derived based on the conservation law of water mass. It is written as follows.

$$F\frac{\partial h_s}{\partial t} = \nabla \cdot [\mathbf{K} \cdot (\nabla h_s + \nabla z)] + q \tag{2}$$

where h_s is the referenced pressure head [L]; t is the time [T]; **K** is the hydraulic conductivity tensor [L/T]; z is the potential head [L]; q is the source and/or sink [L^3/L^3/T]; and F is the water capacity [1/L] given by

$$F = a'\frac{\theta_e}{n_e} + \beta'\theta_e + n_e\frac{dS}{dh} \tag{3}$$

where a' is the modified compressibility of the medium [1/L], θ_e is the effective moisture content [L^3/L^3], n_e is the effectively porosity [L^3/L^3], β' is the compressibility of water [1/L], and S is the degree of saturation. The Darcy's velocity is given by

$$\mathbf{V} = -\mathbf{K} \cdot (\nabla h_s + \nabla z) \tag{4}$$

The interface boundary with surface water flows can be specified head or flux based on flow conditions.

2.3 Interface Conditions

We argue that physics-based coupling should be imposed at the interface boundary. From dimensionality point of view, 1-D and 2-D surface flows are part of 3-D subsurface boundary. An independent subsurface flow model would consider these boundaries as simplified specified head or flux boundaries. However, surface flows become part of the numerical solution in a coupled model, not specified as *a priori*.

For overland flow, the discontinuity at the interface is not justified for most watersheds (Figure 1) and both the continuity of pressure and exchange flux should be applied:

$$h^o = h^s \quad and \quad Q^o = Q^s \quad \Rightarrow \quad I = \mathbf{n} \cdot \mathbf{K} \cdot \left(\frac{\rho_o}{\rho} \nabla h^s + \nabla z \right) \tag{5}$$

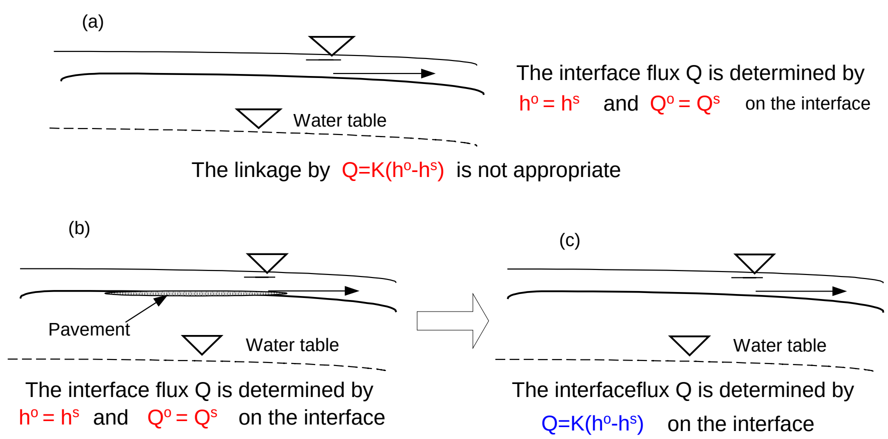


FIGURE 1. Flow interactions between overland regime and subsurface media.

Similarly, the explicit linkage term can be applied to stream-aquifer interaction if field evidence shows that a thin sediment layer exists at the streambed and the subsurface flow domain will not include this sediment layer.

3. NUMERICAL APPROACHES

One major difficulty in implementation of coupling schemes is that the interface conditions cannot be simultaneously imposed, except for the case of using the explicit linkage term. An iterative approach must be taken, thus surface flow solutions are passed to subsurface flow as boundary conditions; in turn, newly computed subsurface solution and boundary flux are feedback to surface flow module as source/sink term. This can also be observed in Equation (5).

Even though it is obvious and straightforward, the surface/subsurface coupling can be summarized as follows (using stream-aquifer interaction for illustration):

$$q = f(h_c, h_s) \tag{6}$$

where q is the exchange flux between stream and subsurface and h_c and h_s are stream water depth and subsurface pressure head.

When a time lagged, decoupled approach is used; the infiltration rate is computed with surface water flow solutions from a previous time step.

$$q^{t+\Delta t} = f(h^t{}_c, h^t{}_s) \tag{7}$$

where Δt is the time-step for subsurface flow. When the iterative coupling approach is adopted, the implicit nature is maintained:

$$q^{t+\Delta t,k+1} = f(h_c^{\ t+\Delta t,k}, h_s^{\ t+\Delta t,k}) \tag{8}$$

where k is the number of nonlinear coupling iteration for subsurface flow. Channel flow can take many sub-time steps in a single subsurface time step and it will be solved within the subsurface nonlinear loop. The explicit linkage term based coupling is only a special case with strict limitation. We will consider the iterative coupling approach as a fully coupled approach with the need of iteration.

4. NUMERICAL EXPERIENTS

Numerical experiments for a series of coupling scheme combinations were conducted to systematically quantify the difference in simulation results. Here only two of them are presented. The continuous interface conditions were imposed for both test examples.

The first example is a simple case of rainfall-runoff process on a horizontal surface of 500 m x 400 m. closed boundary was applied except a 100 m-outlet is located at the center of a 500 m-side (critical flow). The Manning's n value is 0.02. A rainfall intensity of 0.40E-6 m/s was applied for 600 minutes. The underlying porous medium has a vertical saturated hydraulic conductivity of 0.55E-6 m/s and an effective porosity of 0.41. Initially, the water table is 5 m below the land surface. No flow boundary was applied to subsurface domain except the top boundary. The constant time steps of 20 s and 180 s were applied for 2-D overland flow and 3-D subsurface flow, respectively. The 900-minute simulations were performed with the time-lagged decoupled approach and the fully coupled approach and the outflow discharge hydrographs were compared (Figure 2).

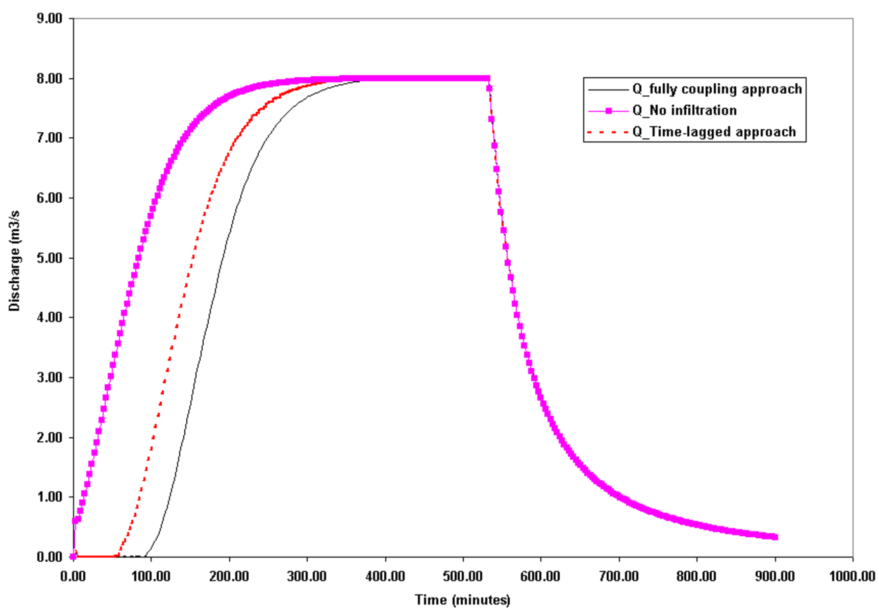


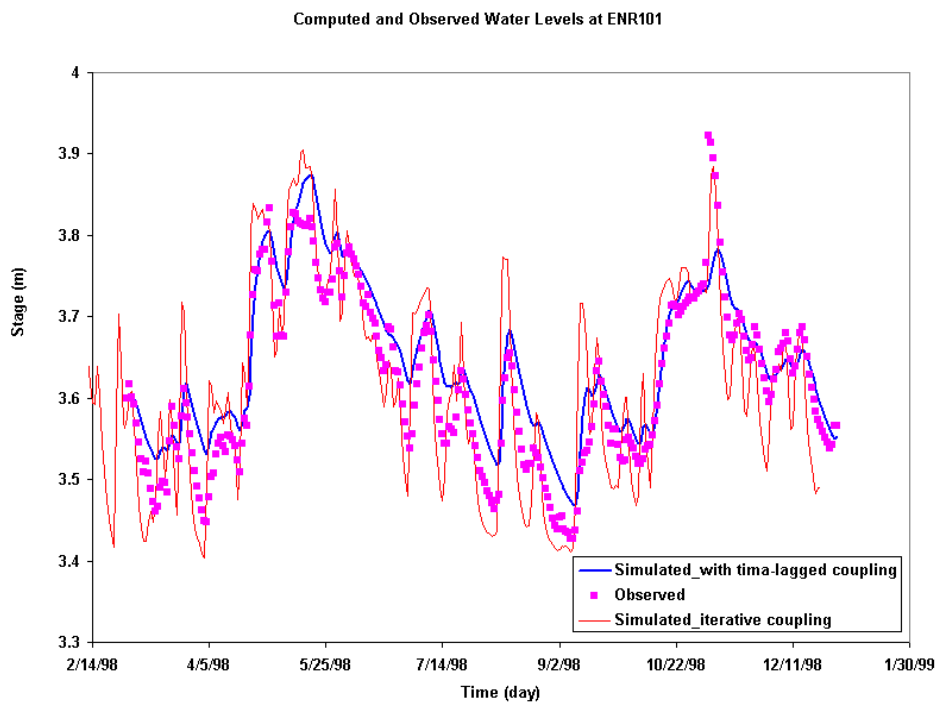
FIGURE 2. Comparison of Outflow Hydrographs

It can be seen from Figure 1 that the time-lagged and the fully coupled approaches resulted in different rising part of the flow hydrograph, after the subsurface domain is fully saturated and the runoff became steady, the simulation results are essentially the same.

The second test example is the surface water and groundwater interaction in a constructed wetland simulated by coupled 2-D overland and 3-D subsurface flow. The details about the study area and model set-up are described in another paper presented at this conference (Huang and Yeh, 2006: WASH123D: PART 5).

The time step for 2-D overland flow is 0.25 hours and in 3-D subsurface flow, the time step of 24 hours was applied. The time-lagged approach produced less accurate results and the fully coupling approach can better capture the flow dynamics and local variation quite well at the two surface water locations (Figure 3).

These simulation results demonstrate that the time-lagged approach could cause significant errors.



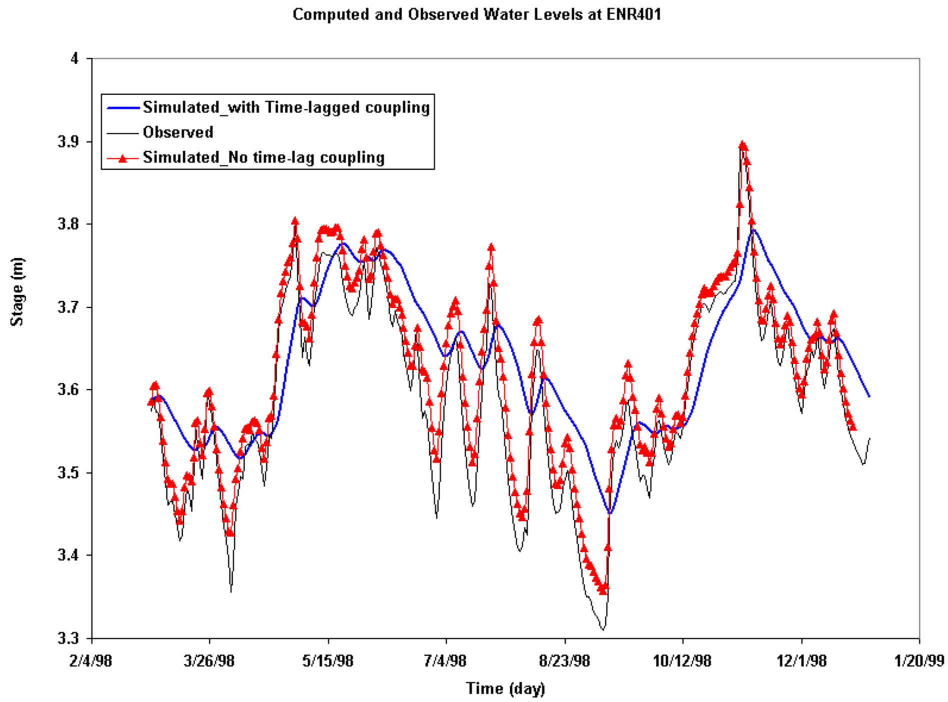


FIGURE 3. Comparison of computed water levels for Example 2.

5. DISCUSSION AND CONCLUSION

By imposing both continuity of state variable and exchange flux, an iterative scheme is needed to implement the coupling process. For Hortonian runoff generation, subsurface flow details are not needed and a time-lagged, decoupled approach may be justified. However, significant error can be possible when this decoupled approach is applied to other flow situations as demonstrated in the test examples.

There may be some numerical advantage on the simultaneous solution of a single global matrix in the linkage-based approach; however, there are also severe limitations: surface and subsurface flow time steps must be the same; evidence and parameter from field observation can support such assumption. It is concluded that different coupling approaches may be justified for flow problems of different spatial and temporal scales and the physical setting of the interface.

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REFERENCES

Abott, M.B., J.C. Bathurst, J.A. Cunge, P.E. O'Connell, and J. Rasmussen, (1986), An introduction to the European Hydrological System- Systeme Hydrologique Europeen, "SHE", 2: Structure of a physically-based, distributed modeling system, Journal of Hydrology, 87, 61-77, 1986.

A.O. Akan and B.C. Yen, (1981), Mathematical model of shallow water flow over porous media. J. Hydraul. Div. 107 HY4 (1981), pp. 479–494.

R.A. Freeze, Role of subsurface flow in generating surface runoff. (1972), 1. Base flow contributions to channel flow. Water Resour. Res. 8 (1972), pp. 609–623.

Gunduz O. and Aral M.M., (2005), River networks and groundwater flow: a simultaneous solution of a coupled system. Journal of Hydrology. 2005; 301: 216-34.

Kollet S.J. and Maxwell R.M.. (2005), Integrated surface-groundwater flow modeling: A free-surface overland flow boundary condition in parallel groundwater flow model. Adv. Water Resour. xx (2005), pp. xxx–xxx.

Morita M, Yen BC. , (2000), Numerical methods for conjunctive two-dimensional surface and three-dimensional sub-surface flows. International Journal for Numerical Methods in Fluids 32: 921–957.

Morita M, Yen BC. (2002), Modeling of conjunctive two-dimensional surface–three-dimensional subsurface flows. Journal of Hydraulic Engineering, ASCE 128: 184–200.

Panday S. and Huyakorn P.S., (2004), A fully coupled physically-based spatially-distributed model for evaluating surface/subsurface flow, Advances in Water Resources, 27 (2004) 361–382

Pinder G.F. and S.P. Sauer, (1971), Numerical simulation of flow wave modification due to back storage effects. Water Resour. Res. 7 1 (1971), pp. 63–70.

Singh V. and S.M. Bhallamudi, (1998), Conjunctive surface–subsurface modeling of overland flow. Adv. Water Resour. 21 (1998), pp. 567–579.

Smith, R. E., and Woolhiser, D. A. (1971). ''Overland flow on an infiltration surface,'' Water Resour. Res., 7(4), 899–913.

Swain ED, Wexler EJ, (1996), A coupled surface-water and groundwater flow model for simulation of stream–aquifer interaction. US geological survey techniques of water-resources investigations, Book 6; 1996. 125 p [chapter A6].

VanderKwaak JE., (1999), Numerical simulation of flow and chemical transport in integrated surface–subsurface hydrologic systems, Doctorate Thesis, Department of Earth Sciences, University of Waterloo, Ontario, Canada.

Yeh, G.T., G. B. Huang, H. P. Cheng, F. Zhang, H. C. Lin, E. Edris, and D. Richards. (2005). A first principle, physics-based watershed model: WASH123D, in Watershed Models. Eds. By Singh VP and Frevert D.K., CRC Press, Boca Raton, FL, September 2005.

Yeh, G.T., G.B. Huang, H. P. Cheng, F. Zhang, H. C. Lin, E. Edris, and D. Richards. (2006). A Numerical Model of Flow, Heat Transfer, and Salinity, Sediment, and Water Quality Transport in WAterSHed Systems of 1-D Stream-River Network, 2-D Overland Regime, and 3-D Subsurface Media (WASH123D: Version 2.0). Technical Report CHL-**-**. Waterways Experiment Station, U. S. Army Corps of Engineers, Vicksburg, MS 39180-6199.