AN INTEGRATED MEDIA, INTEGRATED PROCESSES WATERSHED MODEL – WASH123D: PART 2 – SIMULATING SURFACE WATER FLOWS WITH DIFFERENT WATER WAVE MODELS

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ABSTRACT

The complete Saint Venant equations/two-dimensional shallow water equations (dynamic wave equations) and the kinematic wave or diffusion wave approximations were implemented for one-dimensional channel network flow and two-dimensional overland flow in a watershed model, WASH123D. Careful choice of numerical methods is needed even for the simple kinematic wave model. Since the kinematic wave equation is of pure advection, the backward method of characteristics is used for kinematic wave model. A characteristic based finite element method is chosen for the hyperbolic-type dynamic wave model. And the Galerkin finite element method is used to solve the diffusion wave model. Diffusion wave and kinematic wave approximations are found in many overland runoff routing models. The error in these models has been characterized for some cases of overland flow over simple geometry (e.g. Ponce 1978; Singh 2000 and Parlange 1990). However, the nature and propagation of these approximation errors under more complex two-dimensional flow conditions are not well known. These issues are evaluated within WASH123D by comparison of simulation results on several example problems. The accuracy of the three wave models for one-dimensional channel flow was evaluated with several non-trivial (trans-critical flow; varied bottom slopes with frictions and non-prismatic cross-section) benchmark problems (MacDonnell et al., 1997) and for two-dimensional overland flow with a wetland example. The applicability of dynamic-wave, diffusion-wave and kinematic-wave models to real watershed modeling is discussed with simulation results from these numerical experiments. It was concluded that kinematic wave model could lead to significant errors in most applications. On the other hand, diffusion wave model is adequate for modeling overland flow in most natural watersheds. The complete dynamic wave equations are required in low-terrain areas such as flood plains or wetlands and many transient fast flow situations.

1. INTRODUCTION

Diffusion wave and kinematic wave approximations are found in many overland runoff routing models. The error in these models has been characterized for some cases of overland flow over simple geometry (e.g. Ponce 1978; Govindaraju et al., 1988, 1990; Singh et al., 2005 and Parlange et al., 1990). However, the nature and propagation of these approximation errors under more complex two-dimensional (2-D) flow conditions are not well known.

Parlange et al. (1990) investigated the error in steady state overland flow and demonstrated that in case when kinematic wave is not accurate, diffusion wave is not of much improvement. Singh and coworkers published a series of papers on accuracy of diffusion and kinematic waves on a plane by numerical experiments (Singh et al., 2005). Tayfur et al. (1993) compared numerical solutions of dynamic, diffusive and kinematic wave models for 2-D overland flow on rough surfaces with an average steep slope of 0.086. The results are essentially the same. While diffusion wave approximation is a widely applied model in many current watershed models, kinematic wave approximation is prevailing in many current watershed models in research and practice (for example, HEC-HMS, HSPF, SWMM and some new GIS-based watershed models.) This is attributed to its simplicity and ease of numerical solutions. However, significant error can be possible for kinematic wave.

The complete Saint Venant equations/2-D shallow water equations (dynamic wave equations) and the kinematic wave or diffusion wave approximations were all implemented for one-dimensional (1-D) channel network flow and 2-D overland flow in a watershed model, WASH123D. We will present some numerical experiments on these three options for overland flow and channel flow and demonstrate the potential errors in using one single wave model for all flow situations.

2. GOVERNING EQUATIONS

The governing equations for surface water flows in a watershed can be derived from the conservation of water mass and momentum. The depth-averaged 2-D shallow water equations and the cross-section-averaged 1-D Saint Venant equations are considered accurate representation of 2-D overland flow and 1-D channel network flow, respectively. Only equations for 2-D overland flow are described as follows.

The depth-averaged 2D shallow water equations for overland flow in matrix form are:

$$\frac{\partial U}{\partial t} + \frac{\partial F_x}{\partial x} + \frac{\partial F_y}{\partial y} = G \tag{1}$$

The conservative variables are U=(h, uh, vh); h is water depth; u is the velocity component in the x-direction; v is the velocity component in the y-velocity, respectively. The flux vector F has two components F_x and F_y:

$$F_x = \begin{pmatrix} uh \\ u^2h + \frac{gh^2}{2} \\ uvh \end{pmatrix}, \qquad F_y = \begin{pmatrix} vh \\ uvh \\ v^2h + \frac{gh^2}{2} \end{pmatrix} \qquad G = \begin{pmatrix} R \\ gh(S_{0x} - S_{fx}) \\ gh(S_{0y} - S_{fy}) \end{pmatrix} \tag{2}$$

where R is the source/Sink term as a result of rainfall, evapotranspiration and infiltration, etc. Without the lose of generality, the eddy turbulent term, momentum exchange flux, surface shear stress (wind effect), etc. have been omitted.

The bed slopes are defined as:

$$S_{0x} = -\frac{\partial Z_0}{\partial x}, \quad S_{0y} = -\frac{\partial Z_0}{\partial y} \tag{3}$$

where g is gravitational acceleration and Z_0 is the bed elevation above a datum. The friction slopes can be approximated by the Manning's equation as

$$S_{fx} = \frac{n^2 u\sqrt{u^2 + v^2}}{h^{4/3}}, \quad S_{fy} = \frac{n^2 v\sqrt{u^2 + v^2}}{h^{4/3}} \tag{4}$$

where n is the Manning's roughness coefficient.

The diffusion wave and kinematic wave approximations are based on simplified forms of the momentum equations that ignore the inertial terms. For diffusion wave (DIW):

$$U = (h,0,0) \qquad F_x = \begin{pmatrix} uh \\ \frac{gh^2}{2} \\ 0 \end{pmatrix}, \qquad F_y = \begin{pmatrix} vh \\ 0 \\ \frac{gh^2}{2} \end{pmatrix} \quad G = \begin{pmatrix} R \\ gh(S_{0x} - S_{fx}) \\ gh(S_{0y} - S_{fy}) \end{pmatrix} \tag{5}$$

The water depth gradients are also dropped for the kinematic wave (KIW). It can be easily demonstrated that by combining the continuity equation with simplified momentum equations, the diffusion wave equation is a partial differential equation of parabolic type with one unknown (water depth); on the other hand, the kinematic wave equation is a pure advection equation of water depth with source term.

Boundary conditions for 2-D overland flow are based on governing equations and flow conditions; there can be zero, one or two boundary conditions at the inflow and outflow boundaries.

3. NUMERICAL METHODS

Careful choice of numerical methods is needed even for the simple kinematic wave model. Since the kinematic wave equation is of pure advection, the backward method of characteristics is used for kinematic wave model. A characteristic based finite element method is chosen for the hyperbolic-type dynamic wave model. And the Galerkin finite element method is used to solve the diffusion wave model. The details can be found in (Yeh *et al.*, 2006, 2005) and are omitted here due to page limit. We will focus on how the incorporation of all three optional dynamic wave (DYW), diffusion wave and kinematic wave models can be used to test and verify the applicability and potential errors of simplified DIW and KIW approaches.

4. NUMERICAL EXAMPLES

4.1 Example 1: verification and comparison of steady flow in channels

Three channel flow benchmark problems provided by (MacDonald *et al.*, 1997) were used to verify the numerical schemes implemented for the DYW, DIW and KIW.

The channel is rectangular (Test Problems 1 and 2) or trapezoidal (Test Problem 3) with a length of 1,000 m. A constant flow of 20 m^3/s is applied at the upstream boundary. The flow is pure sub-critical, sub-critical at inflow/super-critical at outflow, and mixed flow with hydraulic jump, respectively for the three tests. The Manning's n value is 0.03. The bed slope is given by an analytical function of the pre-selected water depth. Details about these test problem set-up and the derived analytical solutions can be found in the above-mentioned reference.

The inadequacy and error of the diffusion wave and the accuracy of the DYW solution are demonstrated in Figure 1. The dynamic wave solutions obtained by the characteristics-based finite element method are very accurate and the hydraulic jump in Test Problem 3 was captured. On the other hand, the diffusion wave approximation generated significant errors for the hydraulic jump and an absolute error of less than 5% still exists for the two simpler problems (Figure 1).

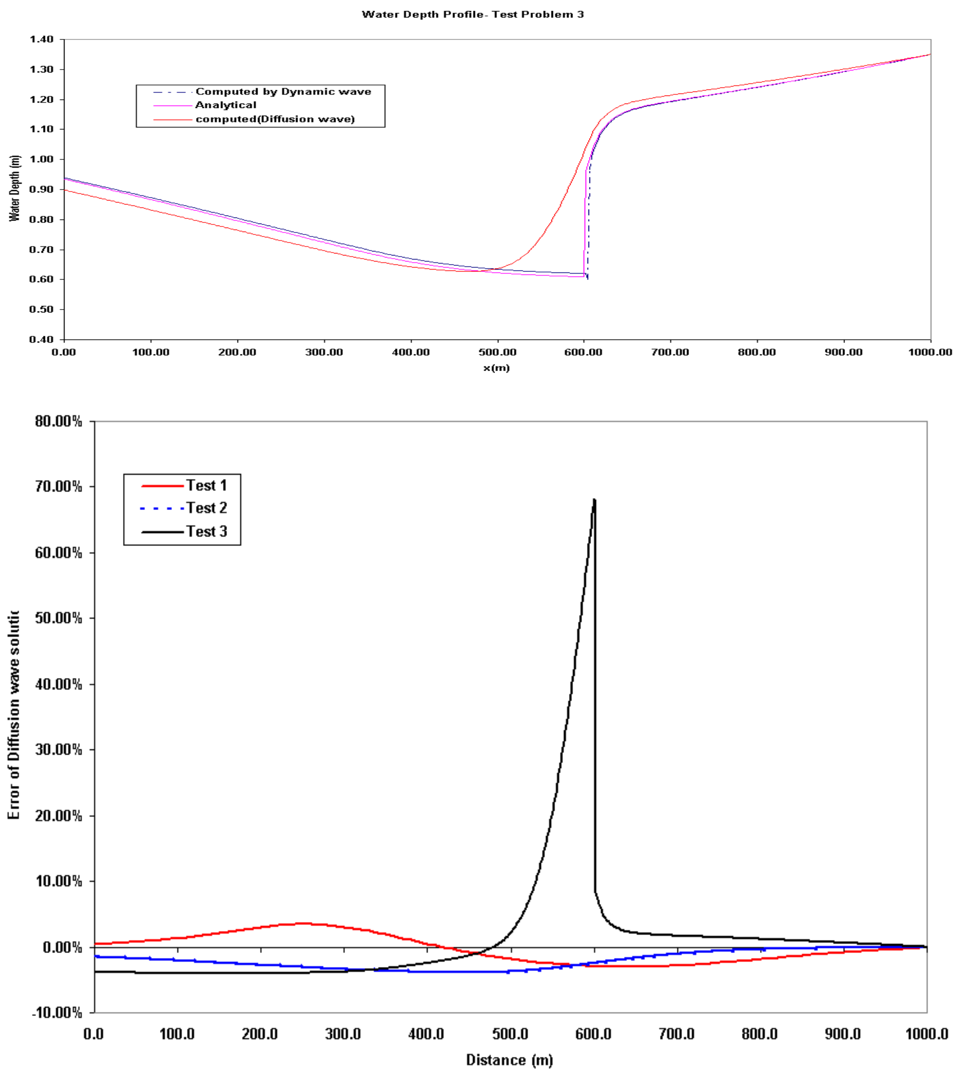

FIGURE 1. Water surface profiles for Test Problem 3 and error of DIW solutions

4.2 Example 2: circular dam break with friction

This example demonstrates the effect of strong inertial terms on significant different results by diffusion wave and dynamic wave.

A circular dam is located on a horizontal smooth surface with a Manning's value of 0.024 (40 m x 40 m). A nominal circular thin wall is located at the circle from the center with a radius of 2.5 m. At the beginning of the simulation, the circular wall has a sudden collapse instantaneously. At time t = 0, the water depth in the dam is 2.5 m, and a water depth of 0.5 m is presented elsewhere (Figure 2). Since the bottom slope is zero, KIW cannot be used. DIW and DYW were applied to simulate the dam break flow process. At the end of one second, the simulation results were compared for transect water depth profiles across the center of the domain and depth hydrograph for the point at the center of the domain (Figures 3 and 4). It can be seen that without consideration of inertial terms, the diffusion wave (DIW) results are incorrect and highly distorted.

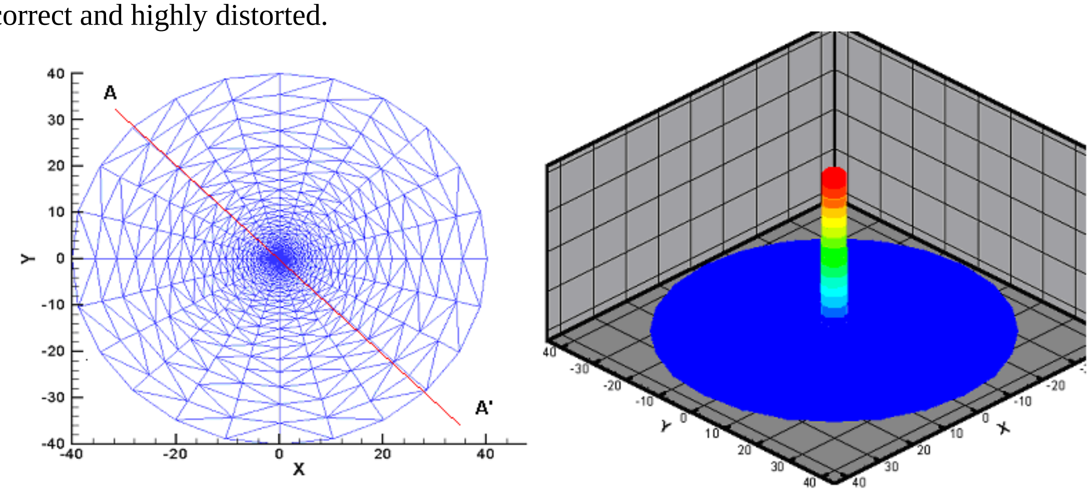

FIGURE 2. Finite element and initial condition for the circular dam problem

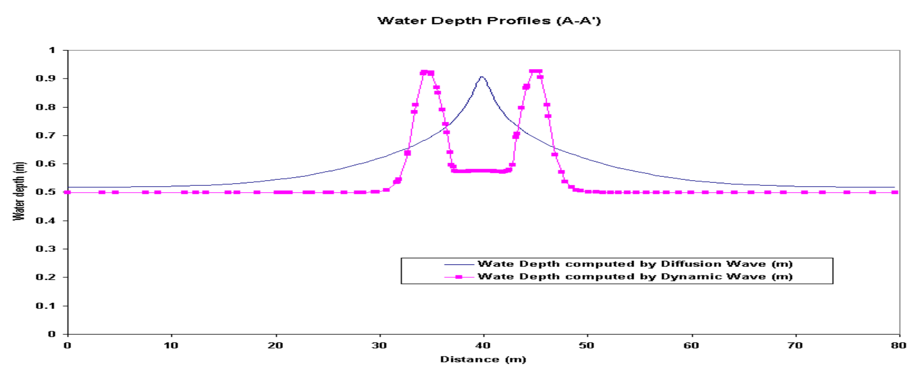

FIGURE 3. Comparison of Computed water depth profile for Example 2.

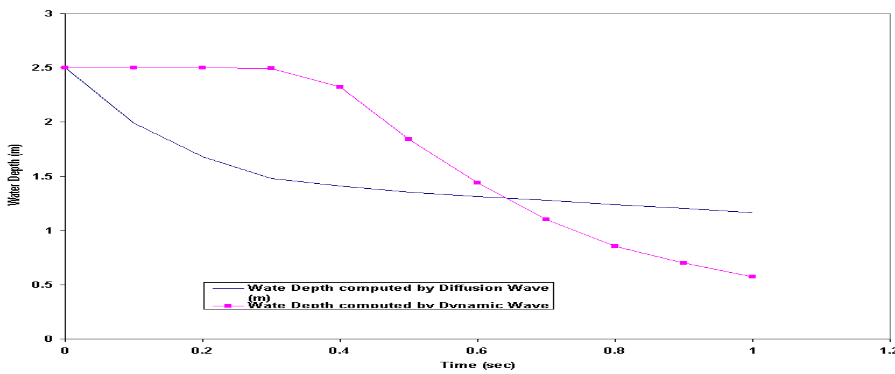

FIGURE 4. Comparison of Computed water depth hydrographs for Example 2.

4.3 Example 3: 2-D unsteady flow in a treatment wetland

One important application of the full two-dimensional shallow water equations is water movement in low terrain areas (e.g., flood plains, wetlands, irrigated farmlands and tidal flats) where simplified diffusion wave approximation may not be appropriate. The numerical simulation of overland flow in a treatment cell in a constructed wetland was made to test model performance of diffusion wave approximation under such a mild slope condition.

The flow domain is about 4.18 km^2. The slope of land surface elevations ranges from 0.0003 to 0.007. The boundary is closed by a perimeter levee, except flow through the inflow and outflow culverts in the levee (Figure 5). There are observed flow and stage data for this treatment cell and a stage monitoring station is located at the center of the cell.

Flow during a time period of 240 hours (10 days) was simulated with time-varied historic flow and stage data (Figure 6) and the simulation results solved by DYW and DIW were compared. Identical finite element mesh, model parameters and boundary conditions were used. Both the DYW and DIW solutions were plotted for the water levels at the observed point with the observed data. Close match (less 0.01 m with water levels range between 3.80 m to 3.95 m) among the observed, DYW and DIW values were obtained (Figure 7). The velocity magnitude and vector plots also show very close pattern. This example demonstrates that DIW can be applicable to slow flow on a very mild slope.

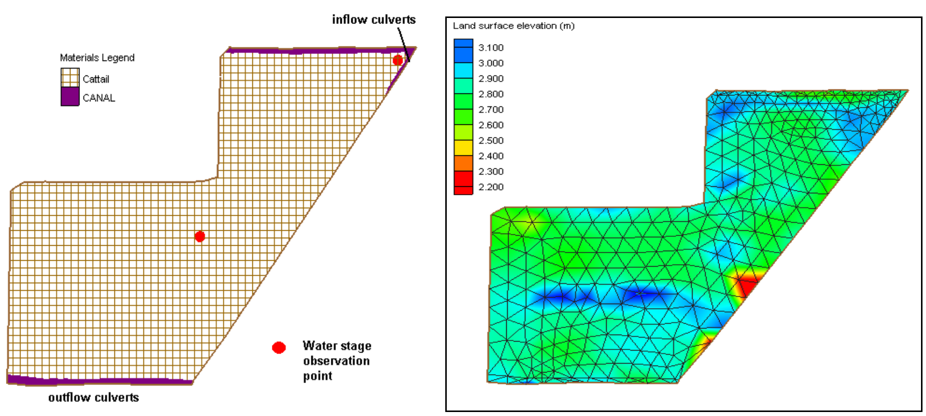

FIGURE 5. Finite element mesh and other model data for Example 3.

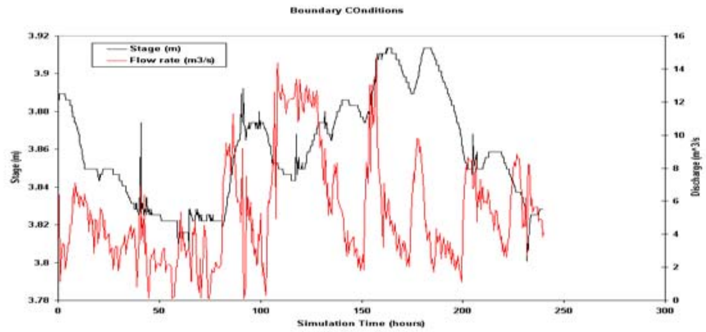

FIGURE 6. Boundary Condition applied in Example 3

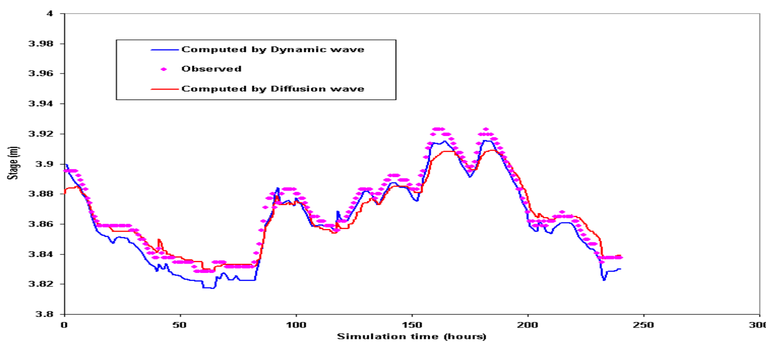

FIGURE 7. Comparison of Observed and Computed Stages for Example 3.

5. DISCUSSION

The dynamic wave model should be mandated for strong dynamic flow cases (e.g., dam break type flow), or when surface wind effect or momentum fluxes are important and where downstream boundaries play a key role (e.g., tidal flow). The diffusion wave model is usually adequate for overland flow on hill-slopes. The accuracy of diffusion wave under very mild slope is not guaranteed as flow dynamics and downstream boundary may play a key role.

The kinematic wave model is still popular in many watershed models. It should be limited to steep-slope mountainous watersheds and where downstream conditions are not important.

An adaptive scheme for applying DYW, DIW or KIW in different sub-domains of a watershed may prove to be efficient and advantageous.

6. CONCLUSION

The implementation of dynamic wave, diffusion wave and kinematic wave models in a single watershed model is an advantage for validation of the approximate models. This is demonstrated by some numerical examples that demonstrate the potential errors in diffusion wave and kinematic wave models for overland flow.

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