DARCY MULTI-DOMAIN APPROACH FOR INTEGRATED SURFACE/SUBSURFACE HYDROLOGIC MODELS

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Abstract

A Darcy multi-domain approach for modelling surface and subsurface hydrologic processes is presented. The diffusive wave approximation is used to model runoff. The resulting equation is formulated as a Darcy nonlinear one. Therefore, the water dynamics in the three physical domains, ground surface, vadose zone and saturated zone, is described through a single Darcy nonlinear equation with domain-dependent parameters. This multi-domain Darcy equation is solved with Mixed Hybrid Finite Element formulation. The time discretisation is implicit and the nonlinear equations are solved within a sequential iterative Picard scheme. This model can describe Hortonian runoff, runoff due to rainfall on saturated areas and seepage. In order to evaluate this new modelling approach different test cases are simulated: runoff on a single slope, preponding and ponded rainfall infiltration in a vertical column and uniform rainfall on non convergent hillslope topography with constant slope. Simulation results show that our model is able to represent saturated area spreading and different runoff generation processes.

1. INTRODUCTION

The main processes governing the hydrological cycle at the catchment scale may be divided in two categories: surface processes and subsurface processes. Surface ones, such as runoff, are strongly coupled to subsurface ones, such as infiltration in the vadose zone or seepage. Therefore, coupled simulations of surface and subsurface flows are necessary for an accurate modelling of real hydrological situations. From a fluid mechanics point of view, these processes can be described as follow: when rainfall occurs, rainfall water can either infiltrate or participate in surface runoff, depending on the soil moisture. If rainfall intensity exceeds what Horton calls the soil infiltration capacity [Horton, 1933], water flows on top of the soil surface and eventually reaches the streamflow. Runoff water can sometimes re-infiltrate where the soil is unsaturated along its flow path. If rainfall intensity is smaller than the infiltration capacity, water infiltrates into the vadose zone to the aquifer and then flows down to the stream through the saturated zone. If the water table is just below the soil surface, the increase of the groundwater level due to infiltration can lead to seepage. In this case, water that has first infiltrated, participates afterwards to surface runoff. Water can also flow at the soil surface if the rain falls on a saturated area or if exfiltration occurs [Kirkby, 1978].

Until recently, hydrologic models did not take into account the coupling between surface and subsurface, essentially for two numerical reasons: (i) runoff kinetics is much faster than the infiltration and groundwater flow kinetics; (ii) for subsurface modelling, runoff dynamics provides the variable boundary condition, both in space and time, at the soil surface [Beaugendre et al., 2004(b)]. In the past, some models proposed at best a weak coupling: depending on the soil properties, a rainfall fraction infiltrated and the other part flowed to the stream by means of a routing procedure [Beven, 2004]. Since a few years, hydrologic modellers are developping models that take into account those processes altogether [Vanderkwaak and Loague, 2001] [Beaugendre et al., 2004(a)] [Panday and Huyakorn, 2004]. The main difference between these models is the way the surface/subsurface (S/SS) coupling is implemented. Some use a first-order law to quantify the exchange terms [Vanderkwaak and Loague, 2001] [Panday and Huyakorn, 2004], others prescribe pressure and velocity continuity at the interface between the runoff model and the model describing soil physics [Beaugendre et al., 2004(a)].

The development of such models is a difficult task because water dynamics is highly nonlinear and the S/SS kinetics are very different. Moreover, the hydrological response is highly dependent on the type of soil. One should mention that the validation of such models strongly suffers from a lack of experimental results and analytical solutions. Nevertheless, two test cases stand today as references in surface/subsurface modelling: an experimental one (artificial rain falling on a sandbox) described in Abdul and Gillham [1984] and a 2D theoretical hillslope system described in Ogden and Watts [2000].

Following the above cited authors, we propose a new model that can take into account all interactions between surface and subsurface. We assume that runoff occurs in a layer at the top of the soil surface. Overland flow is modelled using the diffusive wave approximation, which is considered as a nonlinear Darcy equation. Thus, all surface and subsurface processes are described by a single nonlinear Darcy equation with domain-dependent parameters. In this way, interactions can be treated in a continuous way: pressure and velocity are continuous through the soil surface. Thus, there is no need to introduce a first order S/SS coupling law.

2. PHYSICAL MODEL

2.1. **Overland flow.** For sake of simplicity, the model presented below is written for a 2D hillslope vertical cross-section. The shallow water approximation leads to the mass balance equation [Kirkby, 1978]:

$$\frac{\partial h_r}{\partial t} + \frac{\partial q}{\partial x} = 0 \tag{1}$$

where x is the axis coordinate along the slope, h_r is the water depth and $q = h_r V$ the runoff discharge in which V is the flow velocity. This one is determined with the Manning-Strickler uniform flow formula. Following Kirkby [Kirkby, 1978], V is written:

$$V = \frac{h_r^{2/3}}{n}\sqrt{S} = \frac{h_r^{2/3}}{n\sqrt{S}} S \tag{2}$$

where n is the Manning-Strickler coefficient and S the soil slope.

Combining (1) and (2) leads to the kinematic wave equation. This hyperbolic equation cannot be easily coupled with the Darcy equation, which is a parabolic one. This is why we use the diffusive wave approximation. Assuming that the water depth gradient $\partial h_r/\partial x$ is small compared to the slope $S = \partial(z_{ground})/\partial x$, we can then assume that $S = \partial(z_{ground} + h_r)/\partial x$ and then:

$$V = \frac{h_r^{2/3}}{n\sqrt{S}} \frac{\partial(z_{ground} + h_r)}{\partial x} \tag{3}$$

Using (3), the runoff discharge q equals to:

$$q = \frac{h_r^{5/3}}{n\sqrt{S}} \frac{\partial(z_{ground} + h_r)}{\partial x} \tag{4}$$

Combining equation (1) and (4), we obtain the diffusive wave equation:

$$\frac{\partial h_r}{\partial t} + \frac{\partial}{\partial x}\left(\frac{h_r^{5/3}}{n\sqrt{S}} \frac{\partial(z_{ground} + h_r)}{\partial x}\right) = 0 \tag{5}$$

This equation looks like a generalized Darcy's law with a nonlinear hydraulic conductivity equal to $K_r(h_r) = h_r^{5/3}/n\sqrt{S}$.

2.2. **Unsaturated and saturated flows.** Flows in unsaturated and saturated zones are described by Darcy's law, written as:

$$C(h_s)\frac{\partial H}{\partial t} + \vec{\nabla}.(K(h_s)\vec{\nabla}(H)) = 0 \tag{6}$$

where $H = h_s + z$ is the total hydraulic head, h_s is the capillary pressure, z is the elevation, $C(h_s) = \partial\omega/\partial h_s$ is the soil capillary capacity and $K(h_s)$ is the hydraulic conductivity. In the saturated zone, $K(h_s)$ is constant and in the vadose zone, depends on capillary pressure .

3. NUMERICAL RESOLUTION

The simulation domain is divided in two sub-domains: the runoff layer and the soil, where equations (5) and (6) apply respectively. In both domains, variables have the same meaning and therefore are continuous at the runoff layer/soil interface. The set of flow equations is solved with respect to the total hydraulic head H.

The runoff mass balance equation (5) has to be transformed to account for all S/SS processes and for this equation to be valid for all h values. In the runoff layer, water depth h_r is defined using the total hydraulic head H and the soil elevation z_{ground} : $h_r = H - z_{ground}$. A negative value of h_r means that there is no water in the runoff layer and, consequently, no runoff. In this case, the layer hydraulic conductivity is supposed to be equal to zero. However, we cannot define a hydraulic conductivity equal to zero in our numerical formulation. So, we set the permeability to a residual value ϵ as small as possible for our model to converge. A positive value of h_r means that runoff takes place in the layer. In this case, the horizontal hydraulic conductivity in the runoff layer is equal to $K_r(h_r) + \epsilon$.

In the vertical direction, i.e. in the infiltration direction, we impose a high vertical permeability K_{zz} so that water flows "instantaneously" through the runoff layer into the soil. If rainfall intensity is smaller than the soil infiltration capacity, i.e. in a non Hortonian regime, water flux at the runoff layer/soil interface is equal to the one imposed at the top of the runoff layer. If rainfall intensity is higher than the infiltration capacity, i.e. in a Hortonian regime, flux at the runoff layer/soil interface is equal to the one imposed at the top of the runoff layer until the top of the soil becomes fully saturated. When that happens, a fraction of rainfall water infiltrates due to K_{zz}, while the other fraction stays in the runoff layer and then participates in overland flow. Considering these transformations, runoff equation (5) can be rewritten:

$$C_r(h_r)\frac{\partial H}{\partial t} + \frac{\partial}{\partial x}\left((\frac{h_r^{5/3}}{n\sqrt{S}} + \epsilon)\frac{\partial H}{\partial x}\right) + \frac{\partial}{\partial z}\left(K_{zz}\frac{\partial H}{\partial z}\right) = 0 \tag{7}$$

where $C_r(h_r)$ equals 1 when h_r is positive and 0 when h_r is negative. Combining equations (6) and (7), we obtain for the whole domain, including the runoff layer and the soil domain, a single nonlinear Darcy equation written:

$$\tilde{C}(\tilde{h})\frac{\partial H}{\partial t} + \frac{\partial}{\partial x}\left(\tilde{K_x}(\tilde{h})\frac{\partial H}{\partial x}\right) + \frac{\partial}{\partial z}\left(\tilde{K_z}(\tilde{h})\frac{\partial H}{\partial z}\right) = 0 \tag{8}$$

where, in the runoff layer: $\tilde{h} = h_r = H - z_{ground}$, $\tilde{C}(\tilde{h}) = C_r(h_r)$, $\tilde{K_x}(\tilde{h}) = K_r(h_r) + \epsilon$ and $\tilde{K_z}(\tilde{h}) = K_{zz}$ and in the soil domain: $\tilde{h} = h_s = H - z$, $\tilde{C}(\tilde{h}) = C(h_s)$, $\tilde{K_x}(\tilde{h}) = K(h_s)$ and $\tilde{K_z}(\tilde{h}) = K(h_s)$

This equation is solved using a Mixed Hybrid Finite Element formulation. The time discretisation is implicit and the nonlinear terms are solved within an iterative Picard algorithm [Le Potier et al., 1998]:

$$C(h^{n+1,i})\frac{H^{n+1,i+1} - H^n}{\Delta t} = -\vec{\nabla}.(-K(h^{n+1,i})\vec{\nabla}(H^{n+1,i+1})) \tag{9}$$

where n is the time step index and i the iteration index. The model and this algorithm has been implemented in the finite element code CAST3M (www-cast3m.cea.fr), which is a general computational tool developed at the CEA for mechanics and fluid mechanics applications.

4. VALIDATION-APPLICATION

4.1. **Runoff on a single 1D slope.** We compare our runoff model to the results obtained by Kazezyilmaz-Alhan et al. [2005] who studied numerical formulations for kinematic and diffusive wave approximations. One of their test case is runoff on a 1D impermeable slope: runoff is induced by a 30-minute rainfall over a 183 m long parking lot with a rain intensity of $1.4 * 10^{-5} ms^{-1}$. Manning's coefficient is 0.025 and the slope is 0.0016. We compare the outflow rate at the downstream end of the parking to the one given by the analytical solution of the kinematic wave approximation. Figure 1 shows that our model is in good agreement with the kinematic wave solution. The result of our model is similar to the one published by the authors.

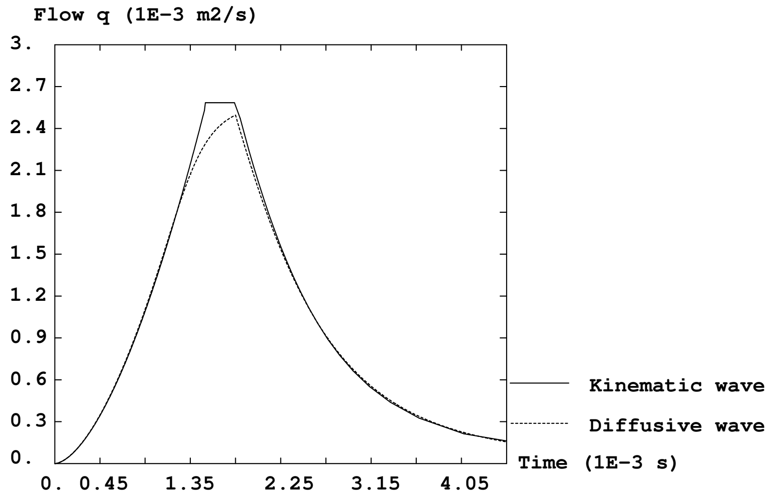

FIGURE 1. Comparison between our model and the analytical solution of the kinematic wave equation

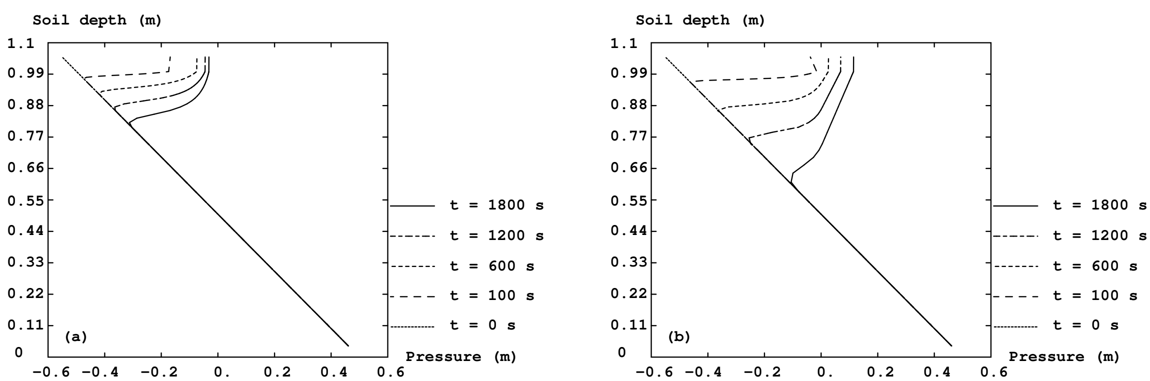

FIGURE 2. Pressure profiles, in meters, at different times in a 1D vertical column for (a) non Hortonian and (b) Hortonian regimes

4.2. **Preponding and ponding infiltration.** Following Rubin [1967], the two 1D vertical infiltration test cases presented here aim to assess if our model can account for both Hortonian and non Hortonian regimes. The column is 1-meter high and the soil is Yolo Light Clay. Van Genuchten's laws for permeability and water content are considered [Van Genuchten, 1980]. In the first case, rainfall is equal to 80 % of the soil infiltration capacity (non Hortonian regime) and in the second, rainfall intensity is 3 times greater than the soil infiltration capacity (Hortonian regime).

Figure 2 shows that, for a non Hortonian regime, the water depth value in the runoff layer is always negative, which means, as expected, that there is no water for runoff. All the rainfall infiltrates.

When rainfall intensity is higher than infiltration capacity, pressure distribution profiles along the column show that: (i) the top of the column becomes saturated, the pressure profile becomes linear; (ii) the water depth in the runoff layer is positive and increases with time, which shows water accumulation in the layer. These results show that a high

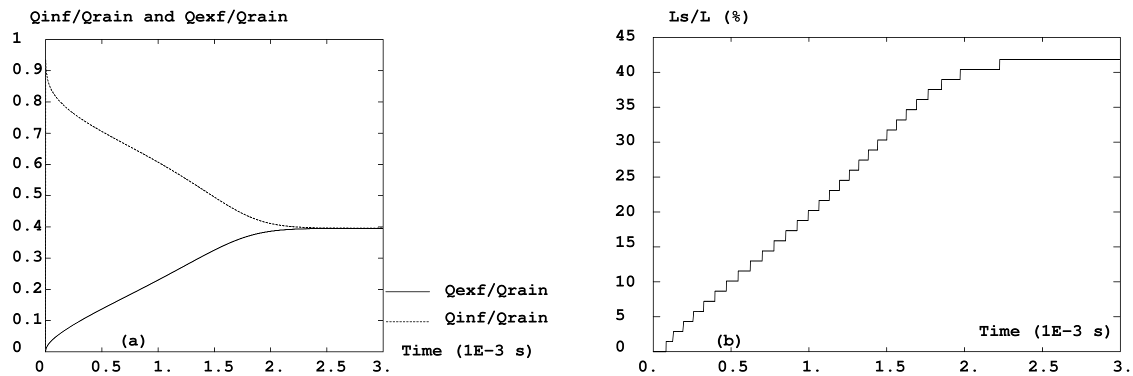

FIGURE 3. (a) Time evolution of the normalized infiltrated flux (dot line) and exfiltrated flux (solid line) and (b) time evolution of the normalized saturated length along the slope for non Hortonian regime

vertical hydraulic conductivity in the runoff layer allows to represent both Hortonian and non Hortonian regimes.

4.3. **Abdul and Gillham's system.** The system described in Abdul and Gillham [1984] is a reference system for modellers interested in S/SS interactions. We use it to validate our model. This experimental system was designed to study the influence of capillary fringe extension on the runoff generation processes. The system is a simple sandbox with no flow boundaries except at the soil surface. Soil surface is thus the only interface that allows water to enter or exit the system, that is what makes this system quite hard to model. Two different Abdul and Gilhham cases are considered: the first one is described in their article and the second one is modified in order to reach a Hortonian regime. The sand box is 1.4 meter wide and from 0.8 to 1 meter high, the slope of the surface is around 0.14. The material is Yolo Light Clay. We assume a Manning coefficient value of 0.1. In the first case, the imposed flux at the top of the domain is 10 % of the soil infiltration capacity and the groundwater table is initially at the toe of the slope. In the second case, the imposed flux is six times greater than the saturated soil hydraulic conductivity and the groundwater table is initially at 0.5 meter from the bottom of the box. To our opinion, this kind of test case has never been tackled.

4.3.1. *Non Hortonian regime.* Figure 3 displays the time evolution of the normalized infiltrated and exfiltrated fluxes through the soil surface and the time evolution of the normalized saturated length along the slope. Fluxes are normalized by the rain intensity and the saturated length by the slope length. The observed dynamics of the system is the following: initially, the groundwater table is at the toe of the slope, but, due to capillarity, the soil is nearly saturated in the whole domain. As a consequence, the rainfall water first infiltrates but a small amount of infiltrated water is enough to make the groundwater level exceed the soil surface elevation at the toe of the slope. The infiltrated water at the top of the slope induces a head gradient that produces exfiltration. Rain which falls on saturated areas is not able to infiltrate anymore and then flows to the outlet in the runoff layer. Then, the infiltrated flux decreases as both the exfiltration flux and the saturated length begin to increase. At the steady state, the saturated length has reached

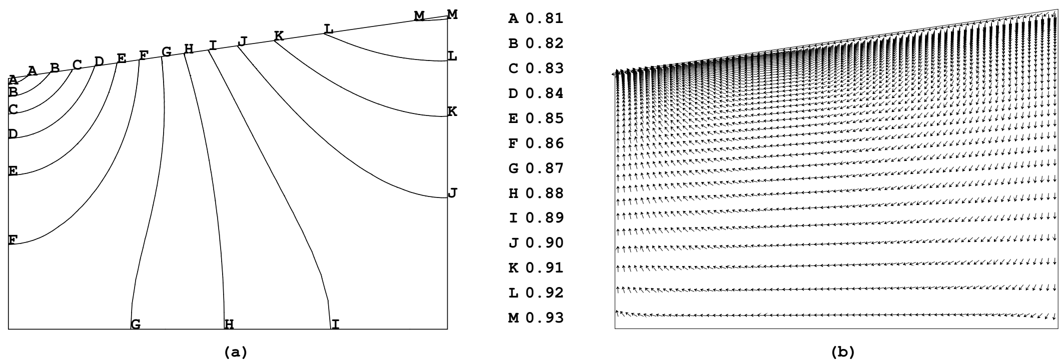

FIGURE 4. Head field, in meters, (a) and normalized velocity field (b) in the domain at steady state for non Hortonian regime

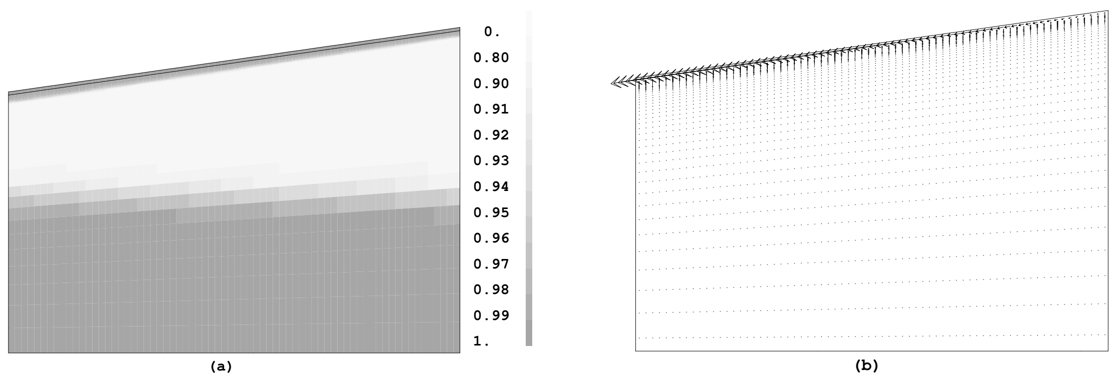

FIGURE 5. Saturation (a) and velocity (b) fields for Hortonian regime

a constant value and the exfiltrated and infiltrated fluxes are equal. We also observe that at early times the infiltrated flux is not equal to 1. This is due to the ϵ residual hydraulic conductivity imposed in the negative water depth areas of the runoff layer. If we want our system to converge, the permeability contrast between dry and wet areas of the runoff layer must not be too large, namely three or four orders of magnitude. Therefore, a small percentage of rainfall flows in the layer instead of infiltrating.

Figure 4 shows the head field and the normalized velocity field in the domain at steady state. We observe the three different surface regimes observed by Abdul and Gillham in their experiments. At the top of the slope, the velocity is vertical and downwards, which means that water is only infiltrating. At the toe of the slope, velocity is vertical and upwards, showing that water is exfiltrating. In the middle of the slope, the velocity vectors are neither vertical, nor horizontal, which means that a fraction of rainfall is infiltrating and the other is flowing on the surface. This velocity field is in good agreement with the one presented in Abdul and Gillham [1984].

4.3.2. *Hortonian regime.* Figure 5 displays both saturation and velocity fields in the domain for a Hortonian regime. The saturation field shows that after a certain time the top of the soil is fully saturated and that water accumulates and flows in the runoff layer, as expected. The discrepancies between velocities in the runoff layer and in the soil illustrate the contrast between runoff and infiltration kinetics.

5. CONCLUSION

The objective of this work is to develop a modelling approach which allows to model the water cycle and the interactions between surface and subsurface processes in a continuous way. We use the diffusive wave approximation to model runoff, and treat this equation as a nonlinear Darcy one. We introduce a runoff layer at the surface in which surface processes are simulated. A single nonlinear Darcy equation with domain-dependent parameters, describing all the S/SS processes, is obtained. This model has been validated with many test cases and this paper presents some of them. We are now tackling 3D situations with slope variability and aim to apply the model to real watershed situations. Nevertheless, a few problems remain, such as those created by the residual runoff permeability or the definition of correct tensorial runoff permeability for steep slopes.

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