Drainage with Unfavorable Viscosity Ratios: a pore-level model study

M. Ferer, Grant S. Bromhal, Duane H. Smith

U.S. Department of Energy,National Energy Technology Laboratory, P. O. Box 880, Morgantown, WV 26507-0880.

The applications of carbon dioxide sequestration in brine-saturated reservoirs, as well as recovery of oil from oil-wet reservoirs, involve the injection of a less-viscous, non-wetting fluid into a porous medium occupied by a more-viscous, wetting fluid: i.e., drainage with an unfavorable viscosity ratio. In these cases, there is a competition between capillary fingering and viscous fingering.

In standard treatments of two-phase flow in porous media, the flow is assumed to be compact, with a uniform residual saturation behind a front, which advances linearly with time. This view of two-phase flow is inconsistent with the cases of fractal capillary fingering at zero capillary numbers, and fractal viscous fingering resulting from injection of an inviscid fluid. Earlier work has shown that when the flow characteristics are not precisely at their fractal limit, the fractal fingering behavior crosses over to standard behavior at a characteristic time which is inversely related to the distance of the viscosity ratio or capillary number from its fractal limit. This earlier work was limited to crossover from one type of fractal fingering to standard flow.

We present results from pore-level modeling for a range of capillary numbers and unfavorable viscosity ratios. The results are analyzed to determine how the competing capillary and viscous fingerings cross over to compact flow. These results are compared with predictions of a scaling hypothesis based upon experience with the aforementioned, simpler fractal-to-compact crossovers from one type of fractal fingering to standard/compact flow.

1.INTRODUCTION

In this paper, we study the crossover from fractal fingering to compact flow for immiscible drainage with unfavorable viscosity ratios, M<1. Clearly, this case is complicated by the competition between fractal capillary fingering and fractal viscous fingering.

For half a century, standard approaches to model flow in porous media have treated two-phase flow as a compact (i.e. Euclidean) process whereby the interface advances linearly with the total amount of injected fluid. This assumed behavior is predicted using saturation-dependent relative permeabilities in an averaged Darcy's law, as in the treatments of Buckley-Leverett or Koval. [Dullien, 1979] For the past quarter century, it has been appreciated that flow in porous media is fractal (i.e. non-Euclidean) in certain well-defined limits, so that the average position of the injected fluid advances faster than linearly with its total amount. [Lenormand et al., 1988],[Blunt and King, 1990], [Meakin, 1998] In the limit of zero viscosity ratio,

$$M = \mu_I / \mu_D = 0 \tag{1}$$

(μ_I is the viscosity of the injected fluid and μ_D of the displaced fluid) the flow exhibits fractal viscous fingering, which is known to be modeled by self-similar, diffusion-limited-aggregation (DLA) fractals.[Lenormand et al., 1988],[Blunt and King, 1990], [Meakin, 1998] In the limit of zero capillary number, where the injection velocity is infinitesimal, V =0 (i.e., quasi-static injection), the flow exhibits fractal capillary fingering, which is known to be modeled by self-similar, invasion percolation with trapping fractals (IPwt). [Meakin, 1998] The definition of the capillary number is

$$N_c = \mu_D V / \sigma , \qquad (2)$$

i.e. the ratio of the viscous forces (viscosity of the displaced fluid times its average velocity, $\mu_D V$) to the capillary forces (proportional to interfacial tension, σ), i.e. Following Fernandez et al. [1991], this use of displaced fluid viscosity, in Eq. 2 , enables us to ascribe observed injection rate effects to capillary number, even at infinitesimal viscosity ratio. [Wilkinson, 1986] IPwt has been widely investigated to determine both its fundamental properties and its predictions for practical problems. [Meakin, 1998] [Blunt, 2001],[Wilkinson, 1986]

In a number of papers, we have studied fractal to compact crossover. For miscible (zero surface tension) injection, we observed a crossover from fractal viscous fingering to compact flow as the viscosity ratio increased from zero.[Ferer et al., 1995] (In these earlier papers, the inverse viscosity ratio was used, i.e., $M_{earlier} = 1/ M$.) For immiscible drainage with favorable viscosity ratios, where the less viscous displaced fluid wets the porous medium and the more viscous injected fluid is non-wetting, we verified a predicted crossover from fractal capillary fingering to compact flow as the capillary number increased from zero. [Wilkinson, 1986],[Ferer et al., 2006] In both these cases, we found that initially the fluid injection was described by the appropriate fractal fingering; but as the fluid advanced, the injection became compact on a time-scale which decreased as distance from the fractal limit increased, i.e. with increasing viscosity ratio or capillary number. Hence, the larger the distance from the fractal limit, the less time it took for the flow behavior to 'cross over' from fractal to compact behavior, so that the only flows that remained fractal were those at exactly the fractal limit.

For miscible flows, this crossover was observed in both two- and three-dimensions. The same crossover was observed in the interfacial width. Also, the behavior of this crossover enabled us to characterize the dependencies of saturation and fractional flow upon the viscosity ratio, in the long-time, compact limit.[Ferer et al., 1995], [Ferer et al., 2006] Furthermore, there is evidence that the power-laws characterizing the dependence of the crossover time upon distance from the fractal limit do not depend upon the detailed structure of the porous medium. For these miscible systems, we found that the same power-law dependence of characteristic time upon viscosity ratio successfully described systems with a log-normal distribution of pore-throat radii and systems with a uniform distribution as well as systems with coordination number four (square lattice) and three (honeycomb lattice).[Stevenson et al., 2006] This suggests that changing the distribution of local permeabilities and the connectivity of the porous medium should not affect the power-law dependence of characteristic time upon viscosity ratio. Furthermore, for immiscible flows, the theoretical arguments that accurately predict the power-law dependence of the crossover from fractal capillary fingering to compact flow are independent of a particular porous medium structure. [Wilkinson, 1986], [Ferer et al., 2006]

For immiscible drainage with zero viscosity ratio, Fernandez predicted a crossover from fractal capillary fingering to fractal viscous fingering as the capillary number, Eq. (2), increased from zero; he also verified the prediction using a specifically tailored pore-level model.[Fernandez et al., 1991] Results from our flow-cell experiments and from our standard pore-level modeling verified these predictions. [Ferer et al., 2004]

The computer code used in this and in previous studies is a standard pore-level model of drainage, which includes capillary, viscous, and gravitational forces. At sufficiently low capillary numbers, we have demonstrated that our model correctly reproduces the zero-capillary number results from IPwt as well as the correct fractal dimension for DLA in the case of miscible flows with zero viscosity ratio. [Ferer et al., 2003] Furthermore, our pore level model results quantitatively agree with experiments in verifying the above-mentioned crossover from fractal capillary to viscous fingering.

2. CROSSOVER FROM CAPILLARY TO VISCOUS FINGERING

Before we address the crossover to compact flow resulting from finite viscosity ratios, we must first revisit the crossover from fractal capillary fingering to fractal viscous fingering for finite capillary numbers at zero viscosity ratios. For fractal fingering in a two-dimensional, rectangular porous medium where zero viscosity fluid is injected with a constant volume flow rate, q, along the side of the medium with width W , the perpendicular length scale of the fractal pattern , x, is related to the amount of injected fluid, m=qt, by a non-Euclidean fractal dimension, D_f,

$$m = qt = AWx^{D_f - 1} \quad , \tag{3}$$

where A is a constant. For small capillary numbers, Fernandez predicted that the early-time pattern would be characterized by the capillary fingering fractal dimension from IPwt, $D_{f,C} \approx 1.89$, but that the flows would begin to exhibit fractal viscous fingering, $D_{f,V} \approx 1.714$, at a characteristic size Λ (or equivalently a characteristic time $\tau = \Lambda^{D_{f,C}-1}$) which varies with capillary number as

$$\Lambda = \tau^{1.13} = N_c^{-0.6} \ , \tag{4}$$

where the numerical value was determined from the fractal dimension for capillary fingering, $1/(D_{f,C} - 1) \approx 1.13$.[Fernandez et al., 1991] For our small systems, the value of the fractal dimension is close to $D_{f,C} \approx 1.89$; however, simulations on very large systems have shown that $D_{f,C} \approx 1.825$.[Sheppard et al., 1999] Figure 1 shows data for times and capillary numbers in the capillary to viscous fingering regime; as predicted by Fernandez, the data are well represented by the functional form

$$X(t, N_c) = t^{1.13} \chi(tN_c^{0.53}) \ . \tag{5.a}$$

Well past crossover, $tN_c^{0.53} >> 1$, this function must exhibit fractal viscous fingering with $1/(D_{f,V} - 1) \approx 1.4$ so that

$$\lim_{tN_c^{0.53} >> 1} X(t, N_c) = \chi_\infty t^{1.13} (tN_c^{0.53})^{0.27} = \chi_\infty t^{1.4} N_c^{0.14} \ . \tag{5.b}$$

The fitting function, given below and shown in Figure (1), has the correct limiting behaviors,

$$\chi(u) = 0.49 e^{-1.38u} + 1.55 u^{0.27} (1 - e^{-1.38u}) \ . \tag{5.c}$$

Knowing the small viscosity ratio behavior in this crossover regime will help in determining the crossover to compact behavior for non-zero viscosity ratios.

In the well-past crossover regime, u>>1, the flows exhibit fractal viscous fingering at all except the smallest times, as can be seen in Figure 2. It is interesting to observe that the breakthrough times (related to near breakthrough saturations of the injected fluid) increase with capillary number while the amplitude B decreases.

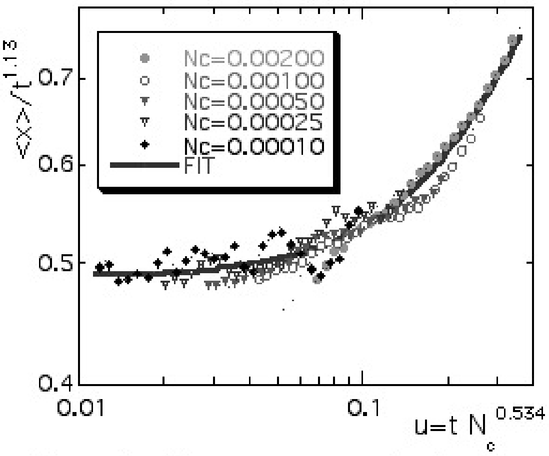


Figure 1 The crossover regime between fractal capillary fingering (small u) and fractal viscous fingering (large u) is shown for viscosity ratio, M = 10^{-4}, and five capillary numbers.

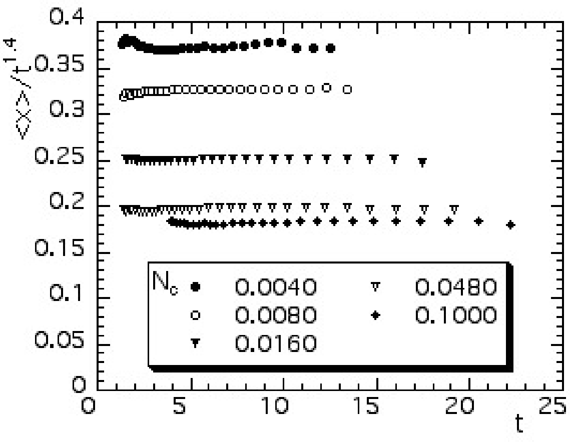


Figure 2 shows the fractal viscous fingering behavior, $<x> = B\ t^{1.4}$ for viscosity ratio, M = 10^{-4}, and for five large capillary numbers. The smallest times are omitted in this figure.

3. CROSSOVER FROM FRACTAL FINGERING TO COMPACT FLOW

Having determined the behavior at viscosity ratios near zero, let us consider how increasing the viscosity ratio changes the flow to compact behavior. We will investigate this effect for three different capillary numbers, one in the crossover regime, N_C=0.0005, one just past crossover, N_C=0.0040, and a third well past crossover, N_C=0.1000.

3.1 Crossover to compact flow for a small capillary number, N_c=0.0005

Figure 3 shows part of one of our porous medium realizations with 7500 pore bodies for a capillary number in the capillary to viscous fingering crossover regime for four increasing viscosity ratios. For the smallest viscosity ratio, the pattern of injected fluid resembles a typical IPwt pattern: exhibiting blockier, denser fingers with significant trapping of the non-wetting fluid. Increasing the viscosity ratio leads to greater occupation of the medium with a more well-defined interface. Visually this evolution is reminiscent of the crossover that we have observed from fractal capillary fingering to compact flow.[Ferer et al., 2006]

Figure 4.a shows data for this capillary number and a range of viscosity ratios. Since these data represent flows in the capillary to viscous fingering crossover regime, the small time behavior is intermediate between the capillary fingering behavior, $< x > \propto t^{1.13}$, and the viscous fingering behavior, $< x > \propto t^{1.4}$. From the fitting function in Eq. (5c), shown in Figure 1, we have a reasonable estimate for the smallest viscosity ratio dependence in this crossover regime. Dividing the data for <x> shown in Figure (4a) by this estimate, we show these data in Figure 4.b. Plotting these data vs. the time divided by a viscosity ratio dependent characteristic time, τ_o all of the data collapse to one curve. This suggests that the function below provides a reasonable representation of these data

$$< x > = X(t, N_c, M) = t^{1.13} \chi(t N_c{}^{0.534}) \mu(t M^{0.25}). \tag{6}$$

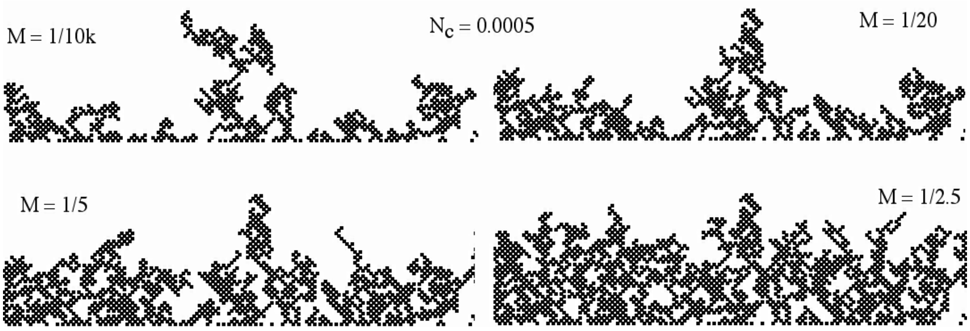
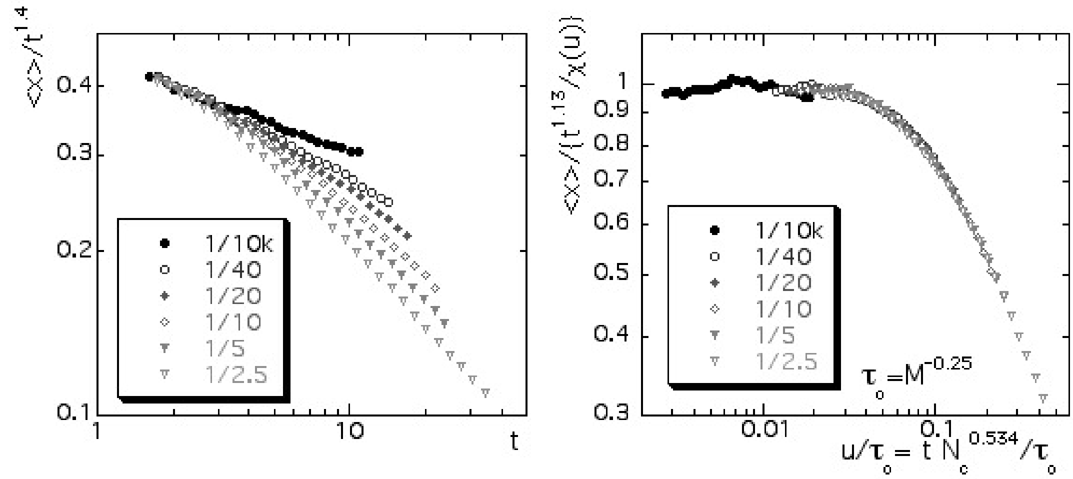


Figure 3 Breakthrough patterns of injected fluid for one of ten realizations for capillary number N_C=0.0005 and for viscosity ratios from M=0.0001 to M=0.4.



Figures 4.a(left) and 4.b(right) The data for the average position of the injected fluid are shown for the capillary number N_C=0.0005 and for viscosity ratios from M=0.0001 to M=0.4.

3.2 Crossover to compact flow for a capillary number just past crossover, N_c=0.0040

As we saw in Figure 2, the crossover from capillary to viscous fingering has occurred for this capillary number at very early times. For the smaller viscosity ratios in Figure 5, there are relatively few fingers with little branching. At this small capillary number, one would expect relatively few fingers because only the largest pores are available given the inlet pressures necessary to displace the defending fluid at the small velocity for this capillary number. One would also expect the fingers to exhibit minimal branching because, as the low viscosity fluid advances, the inlet pressure decreases and, therefore the pressure at the interface decreases, only the largest pores on the interface will be invaded, resulting in minimal branching. As the viscosity of the injected fluid increases, the larger pressures cause more of the pores near the inlet to be invaded leading to more and more fingers. Also at these larger viscosities and pressures, the larger pressure gradients along the fingers result in a smaller pressure near the tip of the longest finger. This decreases the relative growth of the longest finger, leading to fingers of more nearly equal length.

Since, for this capillary number, the crossover to DLA for zero viscosity ratio has occurred at very early times, the function for the average position has the asymptotic form in Eq. (5b), as seen in Figure 6a, where the fractal viscous fingering behavior is clear. As shown in Figure 6b, this data can be collapsed to one curve using a crossover time similar to the one used in our earlier work on miscible viscous fingering.[Ferer et al., 1995] Therefore, for this capillary number, the average position has the form

$$< x >= X(t,N_c,M) = \chi_\infty t^{1.4} N_c^{\ 0.14} \mu((t-1.8/\tau_o)/\tau_o) \qquad (7)$$

where $\tau_o = M^{-0.63}$. Even for the largest viscosity ratios, the "correction term" shifting the time origin, i.e. $1.8/\tau_o$, is a small correction, which does not affect the long time behavior, dominated by t/τ_o. This correction only serves to improve the collapse in the small time regime, v<1.

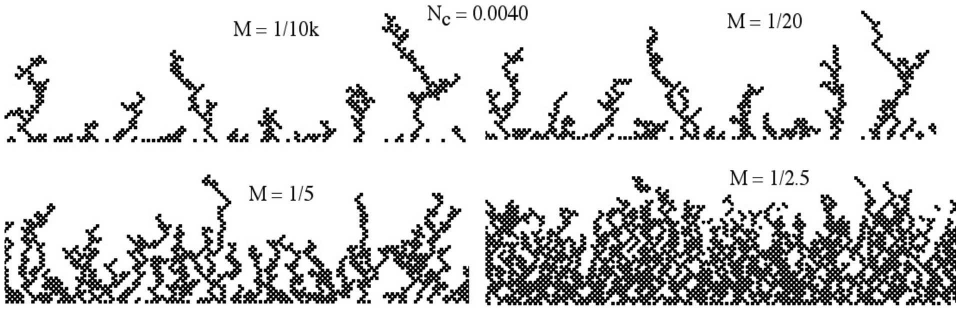
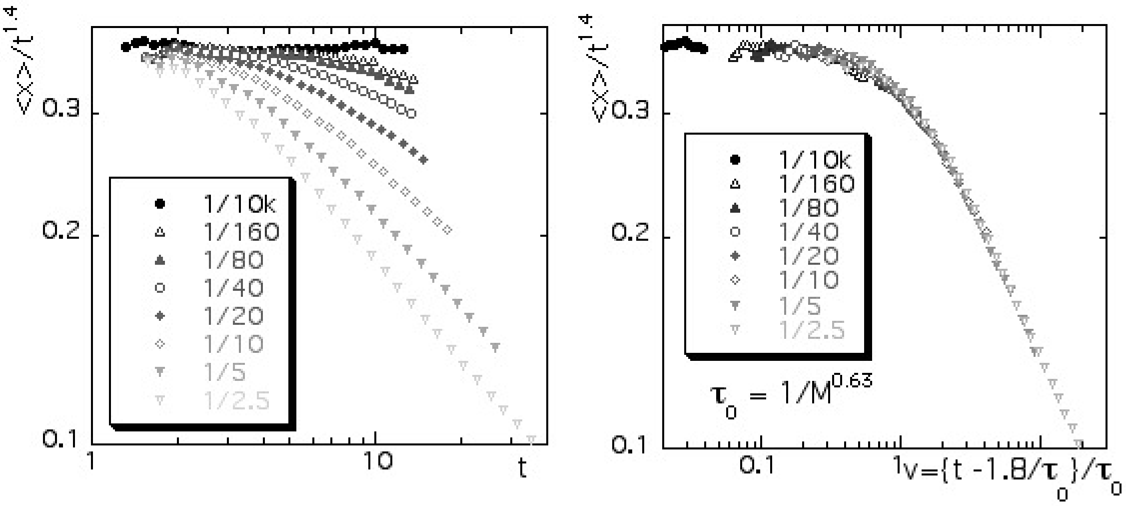


Figure 5 Breakthrough patterns of injected fluid for one of ten realizations for capillary number N_C=0.0040 and for viscosity ratios from M=0.0001 to M=0.4.



Figures 6.a(left) and 6.b(right) The data for the average position of the injected fluid are presented for capillary number N_C=0.0040 and for viscosity ratios from M=0.0001 to M=0.4.

3.3 Crossover to compact flow for a large capillary number, N_c=0.1000

At larger capillary numbers, i.e. larger flow rates, a larger inlet pressure is needed to displace the viscous defending fluid at these flow rates. This leads to invasion of smaller throats at the inlet producing more fingers near the inlet. This also leads to invasion of smaller throats along the fingers producing more side-branching on the fingers, see Figure 7. Increasing the viscosity ratios leads to a collapse of the fractal viscous fingering pattern at smaller viscosity ratios than observed in Figure 5. Indeed the qualitative appearance of these patterns is similar to the appearance of the patterns for the crossover from viscous fingering to compact flow for the miscible cases that we had studied earlier. [Ferer et al., 1995] There is even the same fragmentation of the interface observed in the miscible systems where small fluctuations in the pressure would cause local advances and retreats, fragmenting the interface. It occurs here because the capillary forces are becoming less dominant than the viscous forces.

Again, for this capillary number, the crossover to viscous fingering for zero viscosity ratio has occurred at very early times, so the function for the average position has the asymptotic form in Eq. (5b), as seen in Figure 8a, where the fractal viscous fingering behavior is clear. As shown in Figure 8b, these data can also be collapsed to one curve using a crossover time time similar to the one used in the previous section. Therefore, for this capillary number, the average position has the form

$$< x >= X(t, N_c, M) = \chi_\infty t^{1.4} N_c^{\ 0.14} \mu((t - 4/\tau_o)/\tau_o) \quad (7)$$

where $\tau_o = M^{-0.5}$. Again, the "correction term" shifting the time origin is a small effect which does not change the long time behavior, serving only to improve the collapse in the small time regime, v<2.

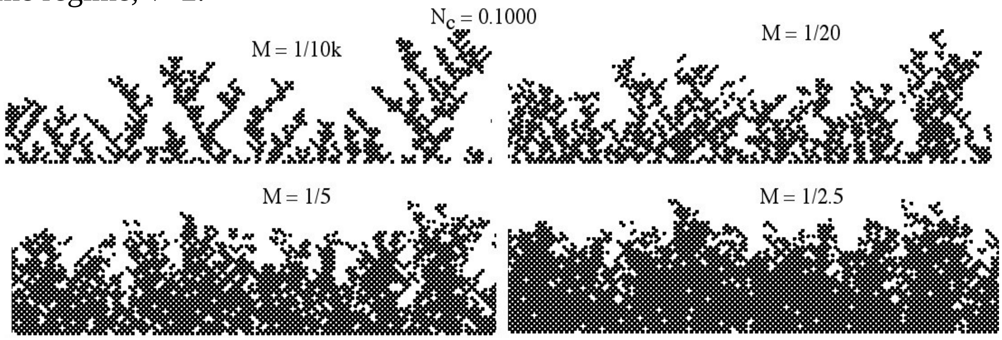
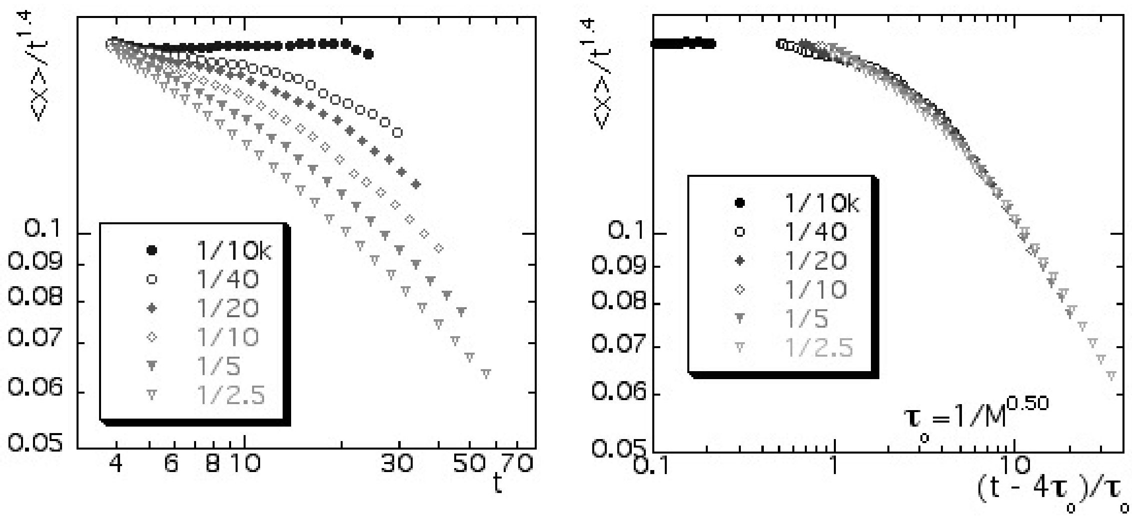


Figure 7 Breakthrough patterns of injected fluid for one of ten realizations for capillary number N_C=0.1000 and for viscosity ratios from M=0.0001 to M=0.4.



Figures 8.a(left) and 8.b(right) The data for the average position of the injected fluid are presented for capillary number N_C=0.1000 and for viscosity ratios from M=0.0001 to M=0.4.

4. CONCLUSIONS

We have presented data that show crossover from fractal fingering to compact flow at a characteristic time, which depends upon capillary number and viscosity ratio. Although our study was restricted to the average position of the injected fluid, our previous work has shown that other examples of fractal-to-compact crossover affect the interfacial width as well as the saturation and fractional flow profiles in ways that can be determined from the behavior of the average position. [Ferer et al., 1995] Furthermore, this earlier work provides evidence that the salient features of our predictions are independent of the porous medium structure. In characterizing the crossover in this paper, we assume that crossover depending upon capillary number can be separated from the crossover depending upon viscosity ratio, as described by the variable-separable equation

$$< x >= X(t,N_c,M) = t^{1.13}\chi(t/\tau(N_c))\mu(t/\tau_o(M)) . \qquad (8)$$

In spite of the excellent agreement between this assumption and the data, as shown in Figures 4b, 6b and 8b , this assumption is flawed because the apparent power law in the viscosity ratio crossover time $\tau_o(M)$ is not independent of capillary number. We are planning further investigation to determine whether this apparent dependence upon capillary number is real or merely an artifact of our power law characterizations of the crossover time.

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