STREAMLINE METHODS ON FAULT ADAPTED GRIDS FOR RISK ASSESSMENT OF STORAGE OF CO_2 IN GEOLOGICAL FORMATIONS

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Abstract

Geological storage of CO_2 in possibly fractured and faulted media, involves the risk of leakage. The extent of leakage may be assessed with statistical methods through analysis of simulations of multiple realizations of a stochastic model. Numerical simulation of numerous such realizations typically requires considerable computational cost, motivating the use of fast numerical methods, such as streamline simulation, for screening. Streamline methods have shown to be effective for reservoir characterization and simulation. In this work we will develop methodology which allows for tracing of streamlines in fractured or faulted media. The work is motivated in part by the need to assess potential of geological storage of CO_2 and is also highly relevant for reservoir simulation.

1. Introduction

Carbon dioxide (CO_2) storage is considered to be a potential key strategy to reduce anthropogenic CO_2 emissions. The principle of carbon dioxide storage is to capture CO_2 produced with the conversion of fossil fuels and sequester the CO_2 in a geological reservoir. Understanding of the risks (see [*Damen et al.*, 2004] for an overview) associated with CO_2 sequestration is one of the key factors affecting public acceptance. The main research topic in risk associated with underground CO_2 sequestration, is leakage. Locally, leakage of CO_2 may be dangerous (in elevated concentrations) to humans, animals and ecosystems, whereas globally, high leakage rates may render the sequestration of CO_2 ineffective as a mitigation option. Typically leakage occurs either as diffuse seepage through the caprock and subsequent overlaying formations, or as concentrated leakage through potentially highly conductive paths such as (abandoned) wells or fractures. From a risk assessment perspective, abandoned wells and fractures form a major challenge since their properties are often at best known statistically [*Celia et al.*, 2004].

Numerical simulation of CO_2 sequestration for risk assessment is typically done in a statistical framework, where evaluation and screening of multiple realizations of a model requires fast numerical methods. Streamline methods have shown to be effective for reservoir characterization and simulation. In this work we will develop methodology which allows for tracing of streamlines in fractured or faulted media. The basis for a streamline method is a sequential splitting of the coupled pressure and saturation equations. A mass-conservative discretization, which handles general faulted grids in a consistent manner, will be used for the pressure equation. In earlier work we have developed streamline tracing on structured and unstructured matching grids. Here we present an extension to grids adapting to faulted media.

Governing equations. For simplicity, the model equations for CO_2 sequestration will be given for two-phase flow, neglecting gravity and capillary pressure. Inclusion of these effects has been discussed elsewhere, e.g. [*Gerritsen et al.*, 2005]. The pressure equation can then be written as [*Settari and Aziz*, 1972]

$$c_t \partial_t p + \nabla \cdot \boldsymbol{q} = b_p, \tag{1}$$

where $\boldsymbol{q}$ is the total velocity (sum of phase velocities), c_t is the total compressibility, and b_p is a source term. Equation (1) is linked to a transport equation for the fluid saturation S

$$\phi \partial_t S + \nabla \cdot \big(\boldsymbol{q} f(S, \boldsymbol{x})\big) = b_s, \tag{2}$$

through Darcy's equation for the volumetric flow density,

$$\boldsymbol{q} = -\lambda(S, \boldsymbol{x}) \nabla p. \tag{3}$$

Here, ϕ, λ, f, and b_s denote porosity, total mobility, fractional flow function, and source terms, respectively. By introducing the time-of-flight [*Batycky*, 1997; *Blunt et al.*, 1996] as an integral along a streamline,

$$\tau(x, y, z) = \int_{s_0}^{s} \frac{\phi}{|\boldsymbol{q}|} ds, \tag{4}$$

the saturation equation, Equation (2), reduces to the one-dimensional hyperbolic equation,

$$\partial_t S + \partial_\tau f(S) = b_s - f(S) \nabla \cdot \boldsymbol{q}. \tag{5}$$

The last term on the right-hand side accounts for compression or expansion of fluids in the case of compressible flows and is identically zero for incompressible flows. Solving the family of one-dimensional problems (5) on a discrete set of streamlines is often much faster than solving Equation (2) over a grid in physical space. Streamlines $\boldsymbol{s} = \boldsymbol{s}(\tau; \boldsymbol{x}_0)$ are defined for a given velocity field $\boldsymbol{q} = \boldsymbol{q}(\boldsymbol{x})$, by

$$\frac{d\boldsymbol{s}}{d\tau} = \boldsymbol{q}(\boldsymbol{x}), \qquad \boldsymbol{s}(0; \boldsymbol{x}_0) = \boldsymbol{x}_0, \tag{6}$$

where τ parameterizes the streamline passing through the starting point $\boldsymbol{x}_0$.

In this work, the domain will be discretized into an unstructured grid, capable of representing faults and fractures. We consider single-phase tracer flow in 2D, where an incompressible version of the pressure equation (1) will be solved on quadrilateral, possibly nonmatching grids, using an MPFA (O-method) [*Aavatsmark*, 2002]. The MPFA-method supplies continuous fluxes on each quadrilateral edge, and a continuously defined velocity field is obtained by interpolation of these edge-fluxes. Tracing of streamlines (integration of (6)) for use in flow simulations has been investigated in [*Hægland et al.*, 2005; *Jimenez* *et al.*, 2005; *King and Datta-Gupta*, 1998; *Matringe*, 2004; *Matringe and Gerritsen*, 2004; *Matringe et al.*, 2005; *Prévost*, 2003; *Prévost et al.*, 2002; *Sun et al.*, 2005].

2. Streamline tracing on faulted grids

2.1. **Introduction: Tracing on Cartesian grids.** To motivate the description of streamline tracing on faulted grids, we start by discussing the basic version on 2D Cartesian grids, which is commonly referred to as Pollock's method [*Pollock*, 1988]. Pollock's method builds a streamline as a series of line segments that each crosses a grid cell in physical space. The segments are constructed such that the exit point of the streamline in one cell is the entrance point in the next cell. By introducing a coordinate transformation, each rectangular grid cell is transformed into a unit square. Linear interpolation of (scaled) edge fluxes is then used to define a velocity field:

$$\boldsymbol{q}^{\mathrm{I}}(x,y) \equiv \begin{bmatrix} F_{x0}(1-x) + F_{x1}x \\ F_{y0}(1-y) + F_{y1}y \end{bmatrix}, \qquad 0 \le x \le 1, \quad 0 \le y \le 1. \tag{7}$$

2.2. **Tracing on faulted grids.**

Solving the pressure equation. For faulted 2D grids we will use a flux continuous control-volume method [*Aavatsmark et al.*, 2001] for the pressure equation. A discrete linear system for cell centre pressures is obtained by assuming a linear potential in each grid cell, and by requiring potential and flux continuity in interaction regions surrounding grid points. The solution procedure will automatically give continuous fluxes on half-edges of each grid cell.

Streamline tracing. Pollock's method cannot be immediately applied to faulted grid cells, since the input is half-edge fluxes. A simple approach may be to add the half-edge fluxes. However, this simple approach causes streamlines to terminate at a no-flow half-edge; see [*Jimenez et al.*, 2005]. This is because adding two half-edge fluxes where one is zero and the other nonzero, will produce a nonzero whole-edge flux, and the information about the non-flow boundary has been lost. A better approach is a subdivision of the cell such that the half-edge fluxes become whole-edge fluxes in the new cells. Mass conservation is used to determine the fluxes on the unknown edges [*Jimenez et al.*, 2005].

3. Streamline tracing for fractured media

Solving the pressure equation. A discretization of a fractured medium domain with volumetric elements in the fractures requires a grid which resolves the geometry of the problem. Due to the small fracture widths, the resulting grid will either consist of a very large number of grid cells or the grid will contain cells with a very large aspect ratio [*Reichenberger et al.*, 2006].

A solution to this is to apply a mixed-dimensional discretization method which realizes fractures as lower-dimensional elements. Solution of the pressure equation for such a model is described in [*Martin et al.*, 2005; *Reichenberger et al.*, 2006].

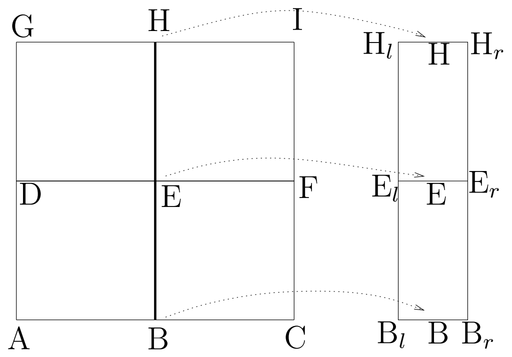
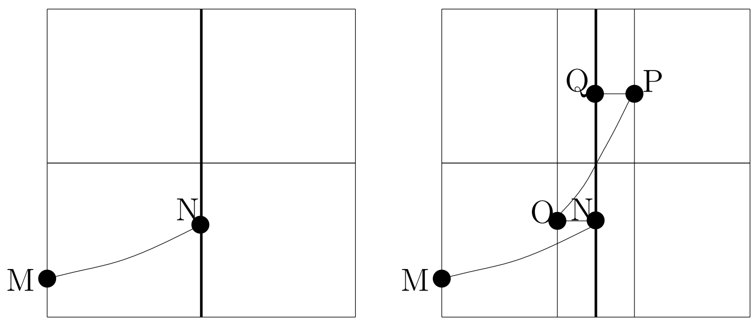


FIGURE 1. Section of a grid with a vertical fracture along (BH) (left). The fracture is associated with a 2D virtual element of width d for computational purposes (right).

Streamline tracing. In a 2D domain, a fracture will be modelled as a 1D object associated with a small width, see Figure 1. By the 2D fracture we will mean the 1D fracture associated with a small width, d, see Figure 1, where the size of d has been exaggerated for illustration purposes. Each edge of the 2D fracture will have an associated flux. Note that horizontal edges for the 1D fracture reduce to a single point in the left part of Figure 1. There is still a flux associated with these edges. Consider tracing a streamline for this simple grid, see Figure 2. A streamline reaching the point N in the fracture, is traced



(a) A streamline enters the grid at the point M, and reaches the fracture at point N.

(b) The streamline tracing through the fracture is done in the 2D fracture.

FIGURE 2. Streamline tracing through a fracture.

through the 1D fracture using the 2D fracture fluxes. The entry point N is associated with a starting point O. The streamline exits the 2D fracture at P, which is associated with a point Q in the 1D fracture. Points O and P are hence pictures in order to calculate exit/entry points of the fracture.

4. Tracer flow modelling

When acid CO_2 is dissolved in water prior to injection in an aquifer for sequestration [*Bachu and Gunter*, 2004], a one-phase miscible displacement will take place. We will assume a simplified model of this displacement, where we consider two fluids, which are completely miscible. The amount of the first fluid contained in the mixture has no influence on the flow of the mixture, hence the name tracer. The volume fraction $C(\boldsymbol{x}, t)$ of the tracer is defined as [*Bastian*, 1999]

$$C(\boldsymbol{x}, t) = \frac{\text{volume of tracer in REV}}{\text{volume of mixture in REV}} \tag{8}$$

Further, we assume that the fluids have the same density ρ. The conservation of mass for tracer can then be modelled by

$$\frac{\partial(\phi\rho C)}{\partial t} + \nabla \cdot \{\rho \boldsymbol{q} C - D\nabla C\} = \rho q_T \tag{9}$$

where the velocity $\boldsymbol{q}$ is the Darcy velocity for the single phase, and D is the hydrodynamic dispersion tensor. Equation (9) is simplified by assuming incompressible flow and neglecting dispersion, which leads to

$$\phi \frac{\partial C}{\partial t} + \boldsymbol{q} \cdot \nabla C = 0 \tag{10}$$

away from sources and sinks.

We observe that the advection equation for the tracer resembles the saturation equation for two phase flow, i.e., Eq. (2). By introducing the time-of-flight relation (4), Eq. (10) simplifies to a one dimensional linear advection equation along the streamlines,

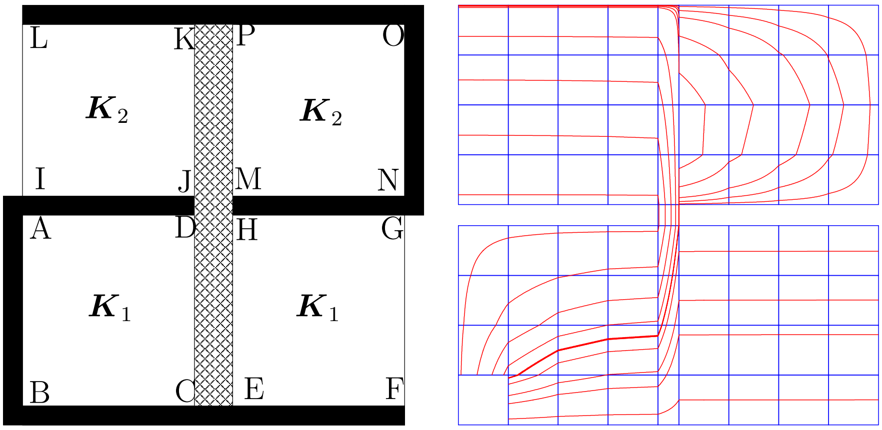
$$\frac{\partial C}{\partial t} + \frac{\partial C}{\partial \tau} = 0. \tag{11}$$

If we use the initial condition $C(0, \tau) = C_0 H(\tau)$ where C_0 is a constant, and $H(\cdot)$ is the Heaviside function, the solution is $C(t, \tau) = C_0 H(\tau - t)$. Tracer flow modelling using streamline simulation has been investigated in [*Crane and Blunt*, 1999].

5. Numerical experiments

We are working on the implementation of a streamline tracing method using a mixed-dimensional finite volume method for the pressure equation. In this paper we simulate the pressure solution of such a method using an MPFA-method.

We consider the test case in Figure 3(a). In that figure the solution domain is the rectangle BFOL, where the heavy black lines are no-flow boundaries. The fracture is the cross-hatched area in middle. A source is located in corner B and edge IL and edge FG are outflow boundaries. We have a lower, upper, and a fracture region, with permeability $\boldsymbol{K}_1 = \boldsymbol{I}$, $\boldsymbol{K}_2 = 10\boldsymbol{I}$, and $\boldsymbol{K}_3 = 1000\boldsymbol{I}$, respectively, where $\boldsymbol{I}$ is the identity matrix, see Figure 3(a). A single-phase pressure equation will be solved on a given discretization of the domain using an MPFA-method, which here, due to a Cartesian grid, reduces to TPFA. The fracture is discretized as a 2D element with a small width d. The solution will give fluxes on each grid cell edge. Some streamlines for this solution is shown in Figure 3(b).

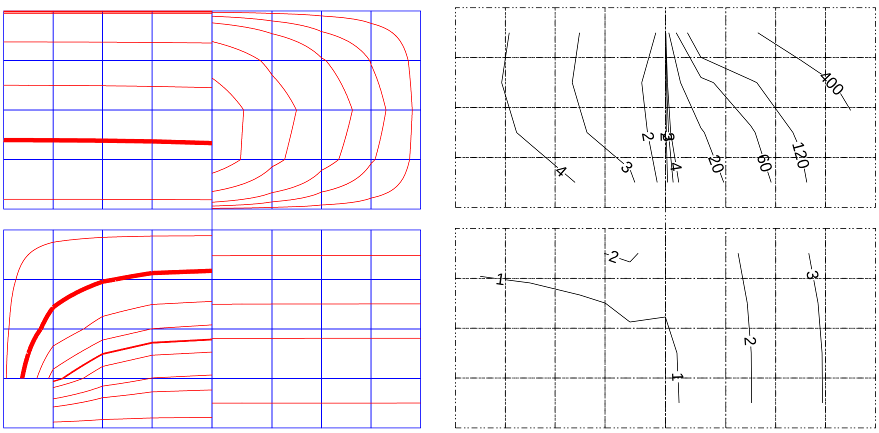


(a) The fracture case.

(b) Streamlines traced for the reference solution

FIGURE 3. Generating a reference solution

Next, the 2D fracture cells are collapsed into corresponding 1D elements, with associated fluxes taken from the 2D solution. Now we apply the streamline tracing method for fractures, see Section 3. The result is shown in Figure 3(b). The heavy streamline



(a) Streamlines traced through a fracture

(b) Tracer arrival time contours

FIGURE 4. Streamline tracing through a fracture

in Figure 4(a) is drawn to indicate where a particular streamline enters and exits the fracture.

Tracer arrival times (see Section 4) are easily computed from the traced streamlines in Figure 4(a). Recall that when tracing streamlines, we can also compute the time-of-flight (TOF) for each grid cell a streamline traverses. If we use the model in Section 4, we get from Eq. (11) that the TOF is equal to the tracer arrival time. Tracer arrival time contours for the case are shown in Figure 4(b).

6. Summary and conclusions

We have described a streamline method for handling flows on irregular grids modelling faulted and fractured media. An important application of this methodology may be risk assessment related to geological storage of CO_2. Potentially this application will require numerous calculations of multiple realizations of geological formations described in a stochastic fashion. Traditional reservoir simulators, based on e.g. finite difference or finite volume methods, require considerable computational cost. In contrast, streamline methods and semi-analytical calculations constitute promising tools for this type of simulations. However, in the case of streamline simulations, this requires that the streamline methods are able to handle potential conduits for leakage like faults and fracture zones.

In this work we have described some preliminary steps in this direction. First, in the case of grids representing faults, special care is needed to handle sections of grid-edges (or faces) representing no-flow conditions, such that streamlines crossing the faults are represented accurately. Second, fractures are usually handled as lower dimensional objects in the computational grid. To determine exit times for streamlines entering a fracture, the fractures are virtually expanded to fully dimensional objects using the measure of the aperture of the fractures. We have illustrated these developments through a numerical experiment.

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