ESTIMATION OF THE WATER TABLE THROUGHOUT A CATCHMENT USING SELF-POTENTIAL AND PIEZOMETRIC DATA IN A BAYESIAN FRAMEWORK

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ABSTRACT

Information about spatial variations in the water table that occur throughout catchments is useful to infer large scale flow patterns, but conventional mapping using piezometric data is invasive, slow, and expensive. Water flow in the subsurface generates an electrical current called the streaming current. The resulting self-potential (SP) (electrostatic) signals can be measured non-intrusively, quickly, and inexpensively at the ground surface. We considered two conceptual models to relate SP signals to the water table. The "infiltration model" relates SP signals to the thickness of the vadose zone, while the "water table model" relates SP signals to the distribution of the water table in unconfined aquifers. These models are first calibrated against field data before a Bayesian method is applied to update a kriged map of the water table obtained from piezometric observations using a kriged SP map. The estimated water tables based on the two conceptual models were combined into one final model using concepts from Bayesian Model Averaging. The method was applied to a small agricultural catchment (~1 km^2) in southern France. The Bayesian framework is useful to avoid over confident predictions when using SP data in hydrogeological estimation because it provides realistic uncertainty estimates.

1. INTRODUCTION

Piezometric data are typically scarce and additional data sources, like non-intrusive geophysical measurements, that provide cheap and reliable information about the water table is in demand. In this study, we estimated the position of the water table and the uncertainty associated with this estimate over a catchment area by combining self-potential (SP) and piezometric data.

The self-potential or spontaneous polarization (SP) method is a passive geophysical method that measures the electrical potential distribution at the ground surface (and possibly in boreholes). The mapping of self-potentials is usually carried out with two non-polarisable electrodes; one is used as a reference, while the other is used to scan variations in the electrical potential at the ground surface. The recorded signals are the superposition of *in situ* signals and telluric currents, which correspond to the main source of noise in addition to cultural activity. Once the noise is removed, there are two main sources of signals. One source of SP signals is associated with ground water flow through the so-called streaming potential or hydroelectric coupling (e.g., Revil et al., 2005). The other source is associated with redox potentials (e.g., Naudet et al., 2004).

The streaming potential dominates over the redox potential in most environments that are not heavily contaminated or contain ore bodies. The microscopic origin of the streaming potential signals lies in ionic processes occurring in the vicinity of the surface of minerals in contact with water. Indeed, the electrical double layer coating the surface of minerals implies the existence of a net excess of electrical charge in the pore water (e.g., Revil et al., 2005). The drag of this—typically positive—excess of charge by the flow of the ground water creates a polarisation of charge at the pore-scale, i.e., the streaming potential, which is one of the cross-coupling effects existing in charged porous materials (Revil et al., 2005). When redox potentials and man-made signals can be neglected or eliminated, the distribution of the measured electrical potentials provides an electrical signature of water flow in the sub-surface (e.g., Fournier, 1989; Aubert and Yéné Atangana, 1996). In this work, we used the first-order approximations of two conceptual models that relate SP signals and the water table.

The objectives of this work were: (1) to illustrate that SP data provide an inexpensive and useful data source in hydrological studies at the catchment scale; (2) to apply a Bayesian estimation method, which uses geostatistical techniques, to integrate SP and piezometric data in order to estimate the water table throughout a catchment. An extended version of this paper has been submitted to Journal of Hydrology and the reader is referred to this coming paper for further details.

2. METHOD

In the last two decades, two potential sources of SP signals associated with ground water flow have been identified. The first conceptual model corresponds to slow infiltration of water through the vadose zone (Aubert and Yéné Atangana, 1996). These authors assumed that SP signals are mainly related to the distance along which water percolates vertically through the vadose zone before reaching the water table. This model will be referred to in the text as the infiltration model. The second conceptual model is related to ground water flow in unconfined aquifers (Fournier, 1989; Revil et al., 2003). According to this model, the strengths of the observed SP signals depend on the contrast of the streaming potential coupling coefficient through the water table (Fournier, 1989) and/or on the contrast of electrical resistivity through the water table (Revil et al., 2003). This model will be referred to in the text as the water table model.

2.1. The infiltration model

If the vadose zone is entirely polarized and if the electrical conductivity distribution in the vadose zone is homogeneous, the resulting self-potential is proportional to the thickness of the vadose zone. Aubert and Yéné Atangana (1996) demonstrated that, to the first-order, the following simple formula can be used to relate SP signals with the elevation of the water table,

$$h(x,y) = z(x,y) - [z_0(x_0,y_0) - h_0(x_0,y_0)] - [\varphi(x,y,z) - \varphi_0(x_0,y_0,z_0)]/c_V, \quad (1)$$

where $\varphi(x,y,z)$ is the electrical potential at a SP station located at the ground surface; c_V is an apparent streaming potential coupling coefficient; $z(x,y)$ and $z_0(x_0,y_0)$ are the elevation of the self-potential station and the reference station, respectively; $h(x,y)$ and $h_0(x_0,y_0)$ are the corresponding elevations of the groundwater table. The infiltration model has been successfully applied to several case studies (e.g., Aubert and Yéné Atangana, 1996) in which the shape of the piezometric surface was confirmed by borehole data.

2.2. The water table model

According to the water table model, SP signals originate from contrasts in the coupling coefficient across the water table (e.g., Fournier, 1989; Revil et al., 2003). A first order approximation that relates SP signals with the water table is (Revil et al., 2003),

$$h(x,y) = h_0(x,y) + [\varphi(x,y,z) - \varphi_0(x,y,z)]/c_W, \tag{2}$$

where $c_W = (C_2 - C_1)$ is an apparent streaming potential coefficient, where C_1 and C_2 are the streaming potential coupling coefficients in the vadose zone and the saturated zone, respectively.

2.3 Bayesian model

Bayes' theorem is an appealing framework for integration of different data types and *a priori* information. When different models are consistent with the available data, it may not be justifiable to rely on a single model. Instead, it might be better to weight the predictions of the various models with weights based on estimated model probabilities. We used concepts from Bayesian Model Averaging (BMA) to condition the posterior probabilities to the two conceptual model types $\mathbf{M} = (M_1, M_2)$, where M_1 corresponds to the infiltration model (see Section 2.1) and M_2 corresponds to the water table model (see Section 2.2).

An estimate of the water table $h(x,y)$ using model M_k can be obtained with Bayes theorem as follows

$$p[h(x,y) \mid \varphi(x,y), M_k] = AL[\varphi(x,y) \mid h(x,y), M_k]\,p[h(x,y)], \tag{3}$$

where $\varphi(x,y)$ is the corresponding estimate from a kriged SP map, A is a normalizing coefficient, $L[\varphi(x,y) \mid h(x,y), M_k]$ is the likelihood of observing $\varphi(x,y)$ given $h(x,y)$ and M_k, $p[h(x,y) \mid \varphi(x,y), M_k]$ and $p[h(x,y)]$ are the posterior and prior probability density functions (pdfs) of the water table, respectively. It should be noted that only collocated estimates are used in Eq. 3, which simplifies the computations without significantly affecting the resulting models.

The prior pdf $p[h(x,y)]$ is obtained by kriging the available piezometric data. The likelihood functions $L[\varphi(x,y) \mid h(x,y), M_k]$ are assumed to have a Gaussian distribution. The likelihood functions are determined by comparing the kriged SP estimates with collocated piezometric observations and estimate c_v (see Eq. 1) and c_w (see Eq. 2), respectively, using linear regression.

The posterior probability that a model is correct given the SP data $p[M_k \mid \varphi(x,y)]$ can be derived by applying Bayes theorem

$$p[M_k \mid \varphi(x,y)] = A_1 L[\varphi(x,y) \mid M_k]\,p[M_k], \tag{4}$$

where $p[M_k]$ is the prior probability that M_k is the correct model type and where the likelihood function $L[\varphi(x,y) \mid M_k]$ is approximated with a scalar by using the standard deviations σ_k around the normal regression models following

$$L[\varphi(x,y) \mid M_k] = \frac{\sum_{l=1}^{K} \sigma_l}{\sigma_k}. \quad (5)$$

In BMA, $L[\varphi(x,y) \mid M_k]$ is evaluated for all possible set of model parameters that the parameterization of the model type can take (Draper, 1995), however, the approximation used in Eq. 5 is sufficient for our needs.

The sum of the prior probabilities $p(\mathbf{m}_k \mid M_k)$ is one:

$$\sum_{k=1}^{K} p(\mathbf{m}_k \mid M_k) = 1. \quad (6)$$

We make the neutral choice that all models are *a priori* equally likely (*Draper*, 1995):

$$p(\mathbf{m}_1 \mid M_1) = p(\mathbf{m}_2 \mid M_2) = 1/2. \quad (7)$$

Finally, the posterior mean $E[h(x,y) \mid \varphi(x,y)]$ and variance $var[h(x,y) \mid \varphi(x,y)]$ are (*Draper*, 1995),

$$E[h(x,y) \mid \varphi(x,y)] = \sum_{k=1}^{K} E[h(x,y) \mid \varphi(x,y), M_k] p(M_k \mid \varphi(x,y)), \quad (8)$$

$$var[h(x,y) \mid \varphi(x,y)] = \sum_{k=1}^{K} var[h(x,y) \mid \varphi(x,y), M_k] p(M_k \mid \varphi(x,y)) + \sum_{k=1}^{K} \{E[h(x,y) \mid \varphi(x,y), M_k] - E[h(x,y) \mid \varphi(x,y)]\}^2 p(M_k \mid \varphi(x,y)). \quad (9)$$

The posterior mean is simply a weighted mean of the different model predictions, but the posterior variance includes an additional term that quantifies the deviations of the individual models from the posterior mean.

3. APPLICATION

3.1. Study site

The Roujan catchment is located in southern France (Fig. 1). It covers a surface area of 91 ha. This catchment was chosen because it is a test site where research on vadose zone and ground water flow has been performed over the last fifteen years (e.g., Voltz et al., 1997; Ribolzi et al., 2000). The site is primarily man-made, with terraced slopes and a major network of ditches collecting the runoff water. Its land use consists mainly of vineyards and shrubs. The soils of the catchment range from clayey to loamy. Most parts of the basin exhibit water tables whose depth fluctuates largely both in space and time. The vadose zone can reach thicknesses of more than 10 meters. The Miocene marine deposits form a layer of marls which can be considered impervious and corresponds to the lower boundary of most groundwaters of the catchment. Presently, 28 piezometers have been installed and the piezometric levels are recorded every week. The electrical conductivity of the ground water is 0.10 S/m (at 25°C).

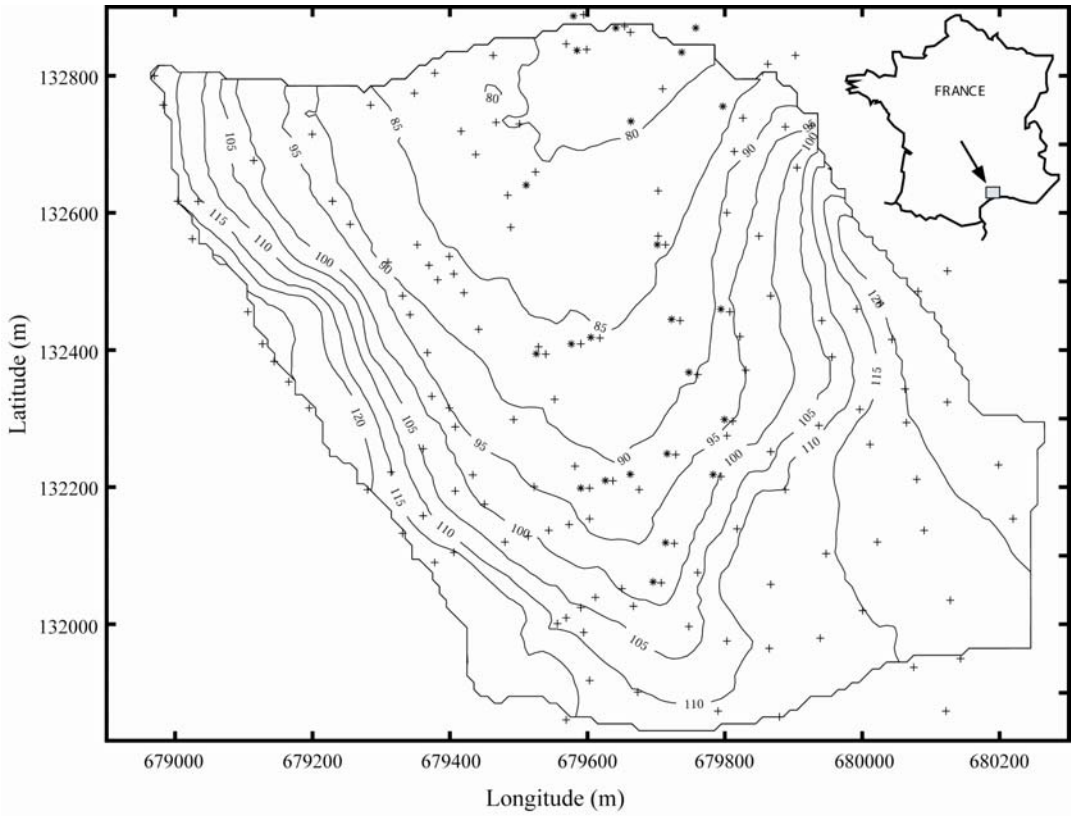

FIGURE 1. Elevation map with positions of piezometers (*) and SP stations (+). The location of Roujan is indicated in the upper right part of the figure.

3.2 Kriging of piezometric and SP data

The SP survey was conducted during two days in June 2005 and it was conducted with Cu/CuSO4 non-polarisable electrodes. We mapped the self potential signals over the entire catchment area with 140 SP stations (see Fig. 1). A Gaussian semi-variogram model with a variance of 90 m^2 and an effective range of 2100 m was used to fit the piezometric data. The SP data were fitted with a nugget of 12 $(mV)^2$ and a Gaussian semi-variogram model with an effective range of 870 m in the north-south direction and 300 m in the east-west direction. Due to the distribution of the piezometers (see Fig. 1), it was not possible to estimate reliable semi-variograms of the piezometric data in the east-west direction. Instead, we assumed the same anisotropy factor as for the piezometric data yielding an effective range of 724 m in the east-west direction. Ordinary kriging was performed to obtain estimates of the water table and the SP signals.

3.3 Likelihood functions

The likelihood functions $L[\varphi(x,y)\,|\,h(x,y),M_k]$ (see Eq. 3) were determined by linear regression. A scatter plot between the measured thickness of the vadose zone and the collocated kriged SP estimates (Fig. 2a) reveals a linear trend with a correlation coefficient of -0.86, where c_v is -0.88 mV/m (see Eq. 1). A scatter plot between the elevation of the water table and the collocated kriged SP estimates (Fig. 2b) reveals a weaker linear trend with a correlation coefficient of -0.58, where c_w is -0.27 mV/m (see Eq. 2). The apparent coupling coefficients c_v and c_w are unusually low. This is probably the consequence of the high electrical conductivity of the ground due to a high cation exchange capacity. For comparison, Revil et al. (2003) and Naudet et al. (2004) reported values of c_w of -3.2 mV/m and -10.6 mV/m, respectively.

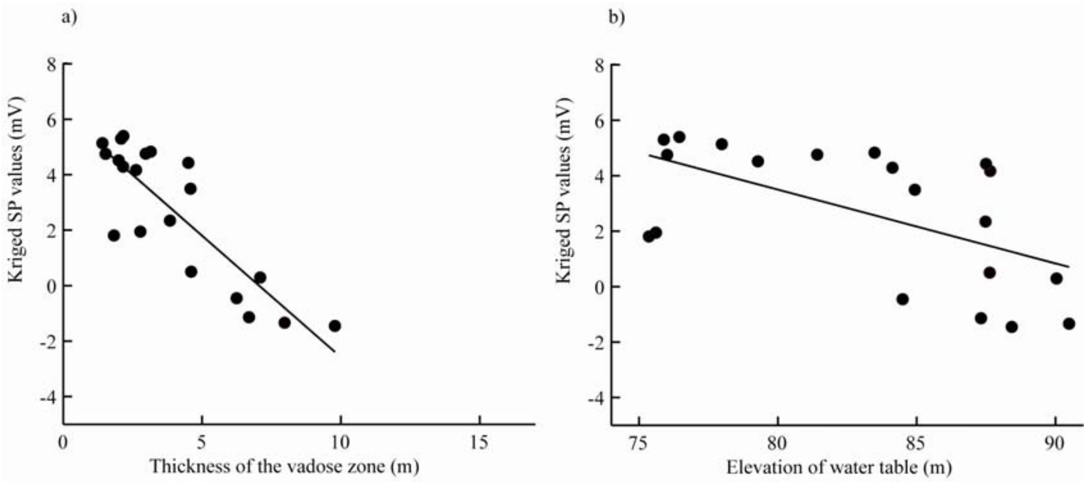

FIGURE 2. Relationship between the self-potential data measured in the field and the water table. (a**)** Correlation between the self-potential data (in mV) and the depth of the water table (in m) (correlation coefficient is -0.86). (b) Correlation between the self-potential data (in mV) and the piezometric heads (in meters above sea level) (correlation coefficient is -0.58).

3.4 Estimation of the water table

For each model (Eqs. 1 and 2) and location, we determine the position of the water table and its uncertainty using Eq. (3). The results are reported in Figs. 3b,c. Finally, the posterior mean of the position of the water table incorporating both models is estimated from Eq. (8) (Fig. 3d). The updated model estimates steep gradients associated with the slopes of the catchment (see elevation map (Fig. 1)), gradients that were not resolved in the prior model (Fig. 3a) because of the distribution of the piezometers. The weighted estimate give a large weight to the infiltration model because of its higher probability of being the correct model.

The estimation uncertainties of the prior model (Fig. 4a), posterior model based on the infiltration model (Fig. 4b), posterior model based on the water table model (Fig. 4c), and the total uncertainty estimated from Eq. 9 (Fig. 4d) are shown. The improvements in the estimation errors for the updated models are most significant away from the piezometers, e.g., the maximum estimation error of the model based on the infiltration model is 2 m compared with more than 8 m for the prior model. The estimation errors are smaller for the models based on the infiltration model compared with the water table model because of the lower variances of the associated likelihood functions. Finally, the weighted models have larger estimation errors because it includes uncertainties regarding the proper model type and is useful to identify areas where the two models deviate significantly.

It should be noted that the self-potential method can be used, with much lower estimation uncertainty, to estimate temporal variations in the water table below a given station. Suski et al. (2005, submitted) found in an infiltration experiment carried out within the Roujan catchment that the relative temporal variations of the piezometric levels could be estimated with a precision of 20 cm when using a fixed network of electrodes.

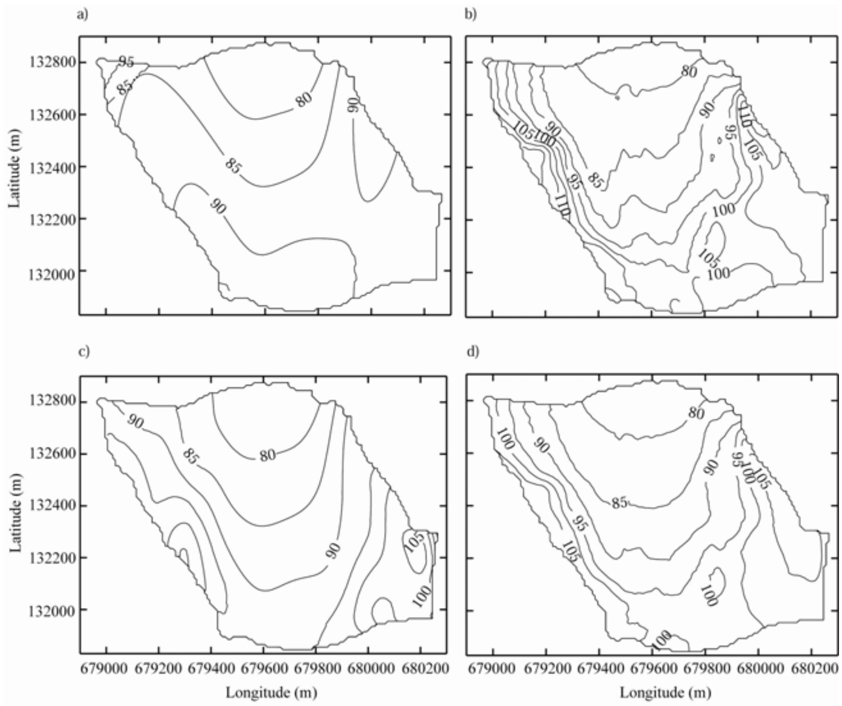

FIGURE 3. The prior model of hydraulic head (a) was updated with the infiltration model (b) and the water table model (c), and averaged using the BMA approach (d).

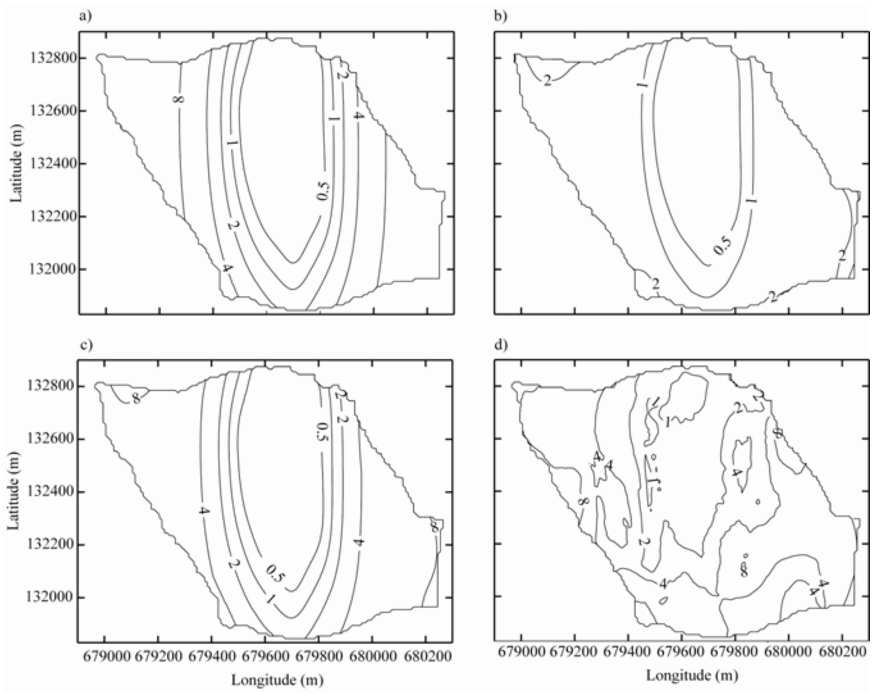

FIGURE 4. The estimation errors of the updated models based on the infiltration model (b) and the water table model (c) are lower compared with the prior model, especially outside of the basin. The weighted estimate of the infiltration model and the water table model has larger uncertainty estimates (d) compared with the individual models (see Eq. 9).

5. DISCUSSION AND CONCLUSIONS

In this work, we estimated the geometry of the ground water table at a catchment scale by integrating SP and piezometric data following a Bayesian approach. The method was applied to data collected at the test site of Roujan, Hérault, in southern France, where 140 SP measurements and 22 piezometric observations were collected in June 2005. We found that the kriged SP data correlated better with the thickness of the vadose zone (correlation coefficient of -0.86) than with the elevation of the water table (correlation coefficient of -0.58); thereby, indicating that the self-potential signal mainly originated from the vadose zone as predicted by the infiltration model by Aubert and Yéné Atangana (1996). Predictions based on both regression models were combined into one final estimate, where the estimation uncertainties also reflect the differences in the predictions between the infiltration and the water table model by Revil et al., 2003. The apparent coupling coefficients c_V=-0.88 mV/m (see Eq. 1)) and c_w=-0.27 mV/m (see Eq. 2) are unusually low at this site and applications at other sites are likely to enjoy higher signal-to-noise ratios and the updated head estimates would be further improved compared with this study.

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