

# ON THE DEFINITION OF MACRO-SCALE PRESSURE FOR MULTI-PHASE FLOW IN POROUS MEDIA

J M. NORDBOTTEN<sup>1,2</sup>, M. A. CELIA<sup>2</sup>, H. K. DAHLE<sup>1</sup>, S. M. HASSANIZADEH<sup>3</sup>

<sup>1</sup>Department of Mathematics, University of Bergen, 5008 Bergen, Norway

<sup>2</sup>Department of Civil and Environmental Engineering, Princeton University, 08544 Princeton, New Jersey, USA

<sup>3</sup>Department of Earth Sciences, Utrecht University, 3508 TA Utrecht, The Netherlands

## ABSTRACT

Micro-scale models have proven to be powerful theoretical tools in groundwater flow and transport modelling. In addition to being useful in estimating traditional parameters, such as (relative) permeability and capillary pressure functions, micro-scale models have recently provided insight into complex multi-phase flow phenomena, such as the so-called dynamic capillary pressure, and are central in investigating theoretical developments in multi-phase flow modelling.

To transfer the results of a micro-scale model to larger scales, a proper definition of macro-scale variables in terms of micro-scale quantities is crucial. One such variable is the pressure. Traditionally, macro-scale pressure of a given phase is defined in terms of the intrinsic phase average; i.e. the average of micro-scale pressure weighted by the volume of the phase. We show, by averaging of micro-scale momentum equations, that the macro-scale pressure in the Darcy equation, which we term the Darcy pressure, is not necessarily the intrinsic phase average of its micro-scale equivalent. This will be the case if there are gradients of porosity or saturation in the system, and these gradients lead to non-negligible changes on the scale of the averaging volume. We have formulated a modified interpretation of macro-scale pressure. The implications of this modification for parameters on the macro-scale are significant, in particular for dynamic relative permeability and capillary pressure. We show that recent interpretations of dynamic capillary pressure can change significantly when this modified definition of macro-scale pressure is used. We also show, through simple example calculations, that inadmissible relative permeability values (e.g. values larger than 1) can result when using the standard average to define macro-scale phase pressures, but that no such problems arise with the new Darcy pressure. These simple calculations also imply that dynamic capillary pressure effects may arise with the standard average, which do not appear with the new pressure definition.

## 1. INTRODUCTION

Pore scale models are a popular tool for predicting flow parameters in macroscopic equations. These models have been successfully applied to predict single phase parameters such as permeability (Dullien 1975; Lindquist 2000), and in terms of understanding more complex multiphase parameters such as relative permeability and capillary pressure.

In particular, network models have been applied to diverse problems such as obtaining supporting evidence for theoretical predictions regarding the functional dependencies of capillary pressure (Reeves and Celia 1996), understanding relationships between micro-scale topology (Sok et al. 2002), and process-based predictions of relative permeability (Øren et al. 1998; Øren and Bakke 2003).

The presence of dynamic effects in both relative permeability and capillary pressure can be argued on intuitive (Barenblatt 1990), experimental (Kalaydjian 1992), and theoretical (Hassanizadeh and Gray 1993a; Hassanizadeh and Gray 1993b; Gray 2000) grounds. The fundamental idea is that when saturation changes occur, it is reasonable to expect that the fluids are distributed away from an equilibrium distribution, thus one may expect that properties related to fluid distribution, such as the relative permeability and capillary pressure, are not at their equilibrium values. Thus it is clear that we can expect the capillary pressure to be a function of the temporal derivative of saturation, along with other measures of change. Restricting our attention to two-fluid flow, we can linearize the dependency on system dynamics,

$$\mathcal{F}^d\left(s, \frac{ds}{dt}\right) = \mathcal{F}^s + \tau(s) \frac{ds}{dt}. \quad (1)$$

Here we have used  $\mathcal{F}$  to denote the functional relationship under consideration, while  $s$  is the wetting fluid saturation and  $\tau$  is the linearization coefficient.

Since the motivation for dynamic terms is inherently linked to the pore-scale, models on this scale have naturally been used to investigate the phenomena, e.g. (Hassanizadeh et al. 2002; Celia et al. 2004; Gielen et al. 2004; Dahle et al. 2005). These investigations indicated that the material parameter  $\tau$  scaled as the length scale squared of the averaging volume in consideration. This motivated further investigations on larger scales (Manthey et al. 2005), and a philosophical discussion of linking relaxation time to the length scale of the system (Celia et al. 2004).

Herein we first motivate the discussion with an example of flow in a tube. In this simple flow regime, the fluid configuration has no dependency on the flow rate. Thus we do not expect any dynamic term as suggested by Equation (1). Yet when averaging over the tube, we observe that a dynamic term appears which scales as length squared. Furthermore, the macroscale relative permeability functions exceed unity and are discontinuous. These counterintuitive results will then be understood as an artefact of the definition of macroscopic pressure (defined as the intrinsic phase average pressure (Slattery 1968; Gray 1975; Marle 1982; de la Cruz and Spanos 1983; Quintard and Whitaker 1988; Hassanizadeh and Gray 1993a; Gray 2000; Hassanizadeh et al. 2002; Celia et al. 2004; Gielen et al. 2004; Dahle et al. 2005; Manthey et al. 2005)). Through careful averaging of the micro-scale flow equations, applying a newly derived averaging theorem (Nordbotten et al. 2006), we will see that the macroscopic pressure in the Darcy equation, which we will term the Darcy pressure, is not the intrinsic phase average pressure. However, the Darcy pressure is related to the intrinsic phase average pressure through a measure of the subscale heterogeneity.

## 2. DEFINITIONS

For a microscale function  $\omega$  and an averaging volume  $V_{\mathbf{x}}$  centered at a point  $\mathbf{x}$ , a general definition of the average of  $\omega$  is:

$$\langle \omega \rangle = \frac{1}{V_{\mathbf{x}}} \int_{V_{\mathbf{x}}} \omega dV. \quad (2)$$

where  $\omega$  may be nonzero over the whole or part of volume  $V_{\mathbf{x}}$ . A special case of interest is when  $\omega$  is defined only over the parts of  $V_{\mathbf{x}}$  that are occupied by a phase  $\alpha$ . This and other definitions are facilitated by the definition of an indicator function  $\gamma_{\alpha}$ , defined by

$$\gamma_{\alpha} = \begin{cases} 1 & \text{if phase } \alpha \text{ exists at } \mathbf{x}, \\ 0 & \text{otherwise.} \end{cases} \quad (3)$$

Let the volume in  $V_{\mathbf{x}}$  occupied by phase  $\alpha$  be defined by  $V_{\alpha, \mathbf{x}}$ . The traditional phase volume average is defined as

$$\langle \gamma_{\alpha} \omega \rangle = \frac{1}{V_{\mathbf{x}}} \int_{V_{\mathbf{x}}} \gamma_{\alpha} \omega dV, \quad (4)$$

while the intrinsic phase volume average is defined as

$$\langle \omega \rangle^{\alpha} = \frac{1}{\langle \gamma_{\alpha} \rangle V_{\mathbf{x}}} \int_{V_{\mathbf{x}}} \gamma_{\alpha} \omega dV. \quad (5)$$

Finally, the fraction of the medium occupied by the  $\alpha$  phase is given by:

$$\epsilon_{\alpha} = \langle \gamma_{\alpha} \rangle \quad (6)$$

We slightly extend this notation, in the sense that we shall assume that we can take intrinsic phase averages over functions  $\omega$  defined only in the phase, even though the right hand side of Equation(5) requires  $\omega$  to be everywhere defined inside the averaging volume. This will have no consequence on the derivation given here.

We will herein restrict ourselves to the consideration of three phases, where subscript  $\sigma$  will denote the solid phase, and the remaining phases are flowing.

## 3. EXAMPLE: FLOW IN A SINGLE TUBE

This section will present a simple example of two fluid flow in a single tube. We will see that despite the simplicity of the problem we arrive at results which are intuitively difficult to accept.

Consider a tube of constant radius and sufficiently large length that the ends of the tube do not affect the solution, see Figure 1. For simplicity of exposition, we will consider two incompressible fluids of equal viscosity, and neglect the width of the fluid interface. We will also neglect the presence of a solid phase outside of the tube, setting  $\epsilon_{\sigma} = 0$ . This implies that the saturation is equal to the phase fraction  $s_{\alpha} = \epsilon_{\alpha}$ . If the flow is horizontal, we can for sufficiently small tubes neglect gravity from our discussion. Then on each side of the interface location  $x_I$  we have single phase flow, which for low enough velocities can be approximated by the Washburn equation

$$q_{\alpha} = -\gamma_{\alpha} \frac{k}{\mu} \frac{\partial P}{\partial x}, \quad \text{for } x \neq x_I. \quad (7)$$

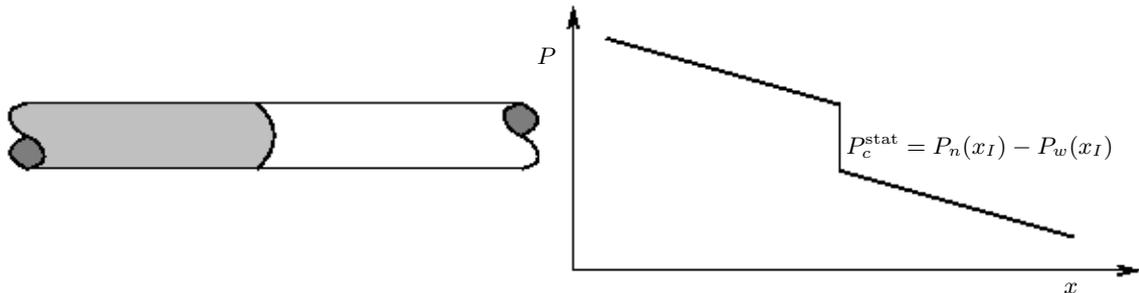


FIGURE 1. In the first figure we see an illustration of the single tube setup, with the non-wetting fluid flowing from left to right. The corresponding pressure distribution is illustrated in the second figure. Note that since we approximate the interface as a point on the  $x$ -axis, the pressure is discontinuous at  $x_I$ .

In this equation, the fluid velocity is denoted  $v$  [ $\text{LT}^{-1}$ ], the inverse resistance to flow is  $k = r^2/8$  [ $\text{L}^2$ ], the fluid viscosity is  $\mu$  [ $\text{FT}$ ], and the pressure is denoted  $P$  [ $\text{FL}^{-2}$ ]. Let us denote the fluid occupying  $x < x_I$  as the non-wetting fluid,  $\alpha = n$ , and the other fluid for the wetting,  $\alpha = w$ . From the incompressibility of the fluids and mass conservation, we have that  $Q = q_n(x) + q_w(x)$  is constant. Observe then from Equation (7) that since only one fluid flows on either side of the interface, we have that  $q_n(x) = Q$  for  $x < x_I$ , and  $q_w(x) = Q$  for  $x > x_I$ . It follows that  $\partial P/\partial x$  is constant for  $x \neq x_I$ .

This system is essentially one dimensional, so the averages defined in Section 2 take the simple form

$$\langle \omega \rangle = \frac{1}{\ell} \int_{x-\ell/2}^{x+\ell/2} \omega dV, \quad (8)$$

where the length of the averaging volume is denoted  $\ell$ . Averaging Equation (7) over an averaging volume we get

$$\langle q_n \rangle = \begin{cases} -\frac{k}{\mu} \frac{\partial}{\partial x} \langle P \rangle^n, & x < x_I - \ell/2, \\ -2\epsilon_n \frac{k}{\mu} \frac{\partial}{\partial x} \langle P \rangle^n, & \text{otherwise} \end{cases}. \quad (9)$$

This follows from the observation that for linearly varying pressure and  $0 < \epsilon_n < 1$ , we can apply Equation (8) to obtain the exact relationship:

$$\nabla \langle P \rangle^n = \frac{1}{2} \left\langle \frac{\partial}{\partial x} P \right\rangle^n \quad (10)$$

Note that the phase fraction  $\epsilon_\alpha$  takes a particularly simple form for this system;  $\epsilon_n = (x_I - x)/\ell + 1/2$ , bounded above and below by 1 and 0, respectively.

We compare Equation (9) with the familiar Darcy's law, which we expect to be valid over a collection of tubes,

$$[q]^\alpha = -k_r(\epsilon_\alpha) \frac{[k]}{\mu_\alpha} \nabla [P]^\alpha. \quad (11)$$

Here brackets indicate macroscale variables. To obtain a mass conservative flow field, it is natural to associate the average flow velocity in the tubes with the Darcy flux ( $\langle q_n \rangle =$

$[q]^\alpha$ ), and define the permeability  $[k] = k$ . We now observe by comparing equations (9) and (11) that if we take the macroscopic pressure to equal the intrinsic phase average pressure ( $[P]^\alpha = \langle P \rangle^n$ ), then the relative permeability function must satisfy

$$k_r(\epsilon_\alpha) = \begin{cases} 2\epsilon_\alpha, & \epsilon_\alpha < 1, \\ 1, & \epsilon_\alpha = 1 \end{cases} \quad (12)$$

This function is discontinuous, and exceeds unity. Both of these observations are in contrast with the expected behaviour (and indeed measurements) of relative permeability.

Related observations can be made with respect to the pressure difference. Let the microscopic pressure jump over the interface due to interfacial tension be denoted  $P_{c,\text{stat}}$ . Then integrating Equation (7) gives

$$\langle P \rangle^n - \langle P \rangle^w = \begin{cases} \text{Undefined} & , \quad \epsilon_n = 0, \\ P_{c,\text{stat}} + \frac{\ell Q \mu}{2 k}, & 0 < \epsilon_n < 1, \\ \text{Undefined} & , \quad \epsilon_n = 1 \end{cases} \quad (13)$$

We can use the time derivative of the expression for  $\epsilon_n$ , together with the relation  $\partial x_I / \partial t = Q$  to write Equation (13) without an explicit dependency on flow rate,

$$\langle P \rangle^n - \langle P \rangle^w = \begin{cases} \text{Undefined} & , \quad \epsilon_n = 0, \\ P_{c,\text{stat}} + \frac{\ell^2 \mu}{2 k} \frac{\partial \epsilon_n}{\partial t}, & 0 < \epsilon_n < 1, \\ \text{Undefined} & , \quad \epsilon_n = 1 \end{cases} \quad (14)$$

Thus at the same time that we introduce counter intuitive relative permeability functions, we also introduce a dynamic term (referring to the dependency on the rate of change of saturation) in the capillary pressure which scales with square of the averaging volume. This dynamic term has the same length squared scaling as has been observed in previous pore scale investigations, as discussed in Dahle et al. (2005).

It is clear from the example, that the dynamic term does not appear as a consequence of the system having a rate-of-change dependent fluid distribution. Indeed, we could let the flow rate go to zero at any time without altering the fluid distribution. Rather the dynamic term as well as the surprising relative permeability function arises as a consequence of our mathematical upscaling. The only arbitrariness in our upscaling was the choice of associating the macroscopic pressure  $[P]^\alpha$  with the intrinsic phase average pressure. This observation motivates the derivation which follows, where we show that the macroscopic pressure in the Darcy equation is not the intrinsic phase average pressure.

#### 4. AVERAGING MICROSCALE EQUATIONS

Many researchers have assumed the validity of Navier-Stokes equations at the pore-scale and have employed volume averaging to obtain Darcy's law, e.g. (Slattery 1968; Gray 1975; Marle 1982; Quintard and Whitaker 1988).

In this section we will give a brief summary of the main points of this derivation, referring the interested reader to literature for detailed accounts. In particular, our derivation herein is closely related to those of Gray (1975) and de la Cruz and Spanos (1983). However, the analysis herein will deviate from previous derivations in the application of a

new averaging theorem for application to the pressure term, following Nordbotten et al. (2006).

The momentum balance equation for a fluid without inertial terms reads:

$$\nabla P - \nabla \cdot \tau - \rho \mathbf{f} = 0. \quad (15)$$

where  $P$  is pressure,  $\tau$  is the stress tensor and  $\mathbf{f}$  represents body forces. When we take the phase average of this equation over an REV for problems with constant density, we get:

$$\langle \gamma_\alpha \nabla P \rangle - \langle \gamma_\alpha \nabla \cdot \tau \rangle - \rho_\alpha \langle \gamma_\alpha \mathbf{f} \rangle = 0, \quad (16)$$

For a Newtonian fluid with dynamic viscosity  $\mu_\alpha$ , the second term can be approximated by assuming the validity of the no-slip condition at fluid interfaces (de la Cruz and Spanos 1983):

$$\langle \gamma_\alpha \nabla \cdot \tau \rangle = -\mu_\alpha \epsilon_\alpha \mathcal{A} (s_\alpha r_r^{\alpha,\alpha} \langle \mathbf{v}_\alpha \rangle^\alpha + s_\beta r_r^{\alpha,\beta} \langle \mathbf{v}_\beta \rangle^\beta) \quad (17)$$

where  $\mathcal{A}$  is a material property accounting for the medium resistance to flow,  $\mathbf{v}$  is the microscopic fluid velocity, the saturation  $s_\alpha$  is defined as  $\epsilon_\alpha / (1 - \epsilon_\sigma)$ , and we have chosen the coordinate system to follow the solid phase. Note that the generalized relative resistivity coefficients  $r_r^{\alpha,\beta}$  appear in this equation, representing the effect of shear forces over the fluid-fluid interface. The generalized relative resistivity coefficients are related to the generalized relative permeability coefficients through

$$\begin{pmatrix} k_r^{\alpha,\alpha} & k_r^{\alpha,\beta} \\ k_r^{\beta,\alpha} & k_r^{\beta,\beta} \end{pmatrix} = \begin{pmatrix} r_r^{\alpha,\alpha} & r_r^{\alpha,\beta} \\ r_r^{\beta,\alpha} & r_r^{\beta,\beta} \end{pmatrix}^{-1}. \quad (18)$$

To obtain a macroscopic equation from Equation (16), we need to have an averaging theorem that allows us to reformulate the average of the gradient of microscopic pressure to macroscopic variables. To this end we introduce the averaging theorem of Nordbotten et al. (2006):

**Theorem 4.1.** *The average of the gradient of a piecewise continuously differentiable function  $\omega$  is related to the gradient of the average of that same function through*

$$\langle \nabla \omega \rangle^\alpha = (\nabla \mathbf{x}^\alpha)^{-1} \cdot \nabla \langle \omega \rangle^\alpha. \quad (19)$$

where  $\mathbf{x}^\alpha = \langle \mathbf{x} \rangle^\alpha$  denotes the centroid of  $V_{\alpha,\mathbf{x}}$ .

We apply this averaging theorem to Equation (16), and obtain

$$(\nabla \mathbf{x}^\alpha)^{-1} \cdot \nabla \langle P \rangle^\alpha - \rho_\alpha g \nabla z = -\mu_\alpha \mathcal{A} (s_\alpha r_r^{\alpha,\alpha} \langle \mathbf{v}_\alpha \rangle^\alpha + s_\beta r_r^{\alpha,\beta} \langle \mathbf{v}_\beta \rangle^\beta) \quad (20)$$

where we have assumed that the only body force acting is gravity, represented in terms of the gradient of the vertical coordinate,  $\mathbf{f} = g \nabla z$ , where  $z$  is taken positive downward. We identify the Darcy flux as the average fluid velocity multiplied by the volume fraction  $\epsilon_\alpha$ , and define the permeability analogously to the single phase case,  $\mathbf{K} = (1 - \epsilon_\sigma) \mathcal{A}^{-1}$  (Nordbotten et al. 2006). We then obtain

$$\frac{\mathbf{K}}{\mu_\alpha} ((\nabla \mathbf{x}^\alpha)^{-1} \cdot \nabla \langle P \rangle^\alpha - \rho_\alpha g \nabla z) = - (r_r^{\alpha,\alpha} \mathbf{q}_\alpha + r_r^{\alpha,\beta} \mathbf{q}_\beta), \quad (21)$$

where we have defined the flux  $\mathbf{q}_\alpha = \epsilon_\alpha \langle \mathbf{v}_\alpha \rangle^\alpha$ .

Now, the the macroscopic formulation of Darcy's law taking into account viscous coupling has the form (Muccino et al. 1998)

$$\frac{\mathbf{K}}{\mu_\alpha}(\nabla[P]^\alpha - \rho_\alpha g \nabla z) = - (r_r^{\alpha,\alpha} \mathbf{q}_\alpha + r_r^{\alpha,\beta} \mathbf{q}_\beta), \quad (22)$$

Comparison of Equation (21) to Equation (22) suggests that the gradient of the macroscopic phase pressure should be related to the gradient of the intrinsic phase average pressure through the relationship

$$\nabla[P]^\alpha = (\nabla \mathbf{x}^\alpha)^{-1} \cdot \nabla \langle P \rangle^\alpha. \quad (23)$$

We will term the macroscopic pressure  $[P]^\alpha$  defined in Equation (23) the *Darcy pressure*, to emphasize that this is the pressure driving flow in the Darcy equation. Equation (23) is the same as the result reported for single phase flow in Nordbotten et al. (2006). There they also derive an approximate relationship between the Darcy pressure and the intrinsic phase average pressure, which is equally applicable to multiphase flow:

$$[P]^\alpha = \langle P \rangle^\alpha - (\mathbf{x}^\alpha - \mathbf{x}) \cdot (\nabla \mathbf{x}^\alpha)^{-1} \nabla \langle P \rangle^\alpha. \quad (24)$$

Concluding this section, we revisit the example from Section 3. Since there is no flow parallel to interfaces in a single tube, we will set the interfacial coupling terms  $r_r^{\alpha,\beta}$  equal to zero for  $\alpha \neq \beta$ . Therefore we will use the multi-phase extension of Darcy's law given in Equation (11) instead of the more general Equation (22). The discussion above suggests that the correct interpretation of macroscopic pressure is the Darcy pressure related to the microscopic pressure through Equation (23). For the single tube, we can easily calculate the gradient of the phase centroid:

$$\frac{\partial}{\partial x} \langle x \rangle^\alpha = \begin{cases} \text{Undefined,} & \epsilon_\alpha = 0, \\ \frac{1}{2} & , \quad 0 < \epsilon_\alpha < 1, \\ 1 & , \quad \epsilon_\alpha = 1 \end{cases} \quad (25)$$

We now insert Equation (25) into Equation (9) and use (23) to obtain

$$\langle q_n \rangle = -\epsilon_n \frac{k}{\mu} \frac{\partial}{\partial x} [P]^n \quad (26)$$

An entirely similar expression can be obtained for the wetting phase. As in Section 3 we compare Darcy's law as stated in Equation (11) with Equation (26) to obtain the macroscopic relative permeabilities

$$k_r(\epsilon_n) = \epsilon_n. \quad (27)$$

Likewise, the expression for the difference between the centroids of the averaging volume and the phase required in Equation (24) can be calculated for the single tube as

$$\langle x \rangle^\alpha - x = \frac{\ell}{2}(\epsilon_\alpha - 1). \quad (28)$$

This allows us to calculate the difference between the macroscopic pressures using equations (28) and (24) in Equation (14), together with the solution of Equation (7):

$$[P]^n - [P]^w = \begin{cases} \text{Undefined,} & \epsilon_n = 0, \\ P_{c,\text{stat}} & , \quad 0 < \epsilon_n < 1, \\ \text{Undefined,} & \epsilon_n = 1 \end{cases} \quad (29)$$

Note that we also have for  $\epsilon_\alpha > 0$ ,  $\partial[P]^\alpha/\partial x$  is constant, such that the Darcy pressure behaves similarly to the microscale pressure.

## 5. DISCUSSION

In the above, we have argued that the natural interpretation we associate with the constitutive relationship relative permeability implies that the macroscopic pressure driving flow in Darcy's law is different from the phase averaged pressure. We have further shown through an example at the microscale that this macroscopic Darcy pressure also leads to a capillary pressure, which for this case, does not have any dynamic dependency.

This leads us to two complementary formulations for our macroscopic system. We can either follow the approach tacitly adopted in many of the references (Slattery 1968; Gray 1975; Marle 1982; de la Cruz and Spanos 1983; Quintard and Whitaker 1988; Hassanizadeh and Gray 1993a; Gray 2000; Hassanizadeh et al. 2002; Celia et al. 2004; Gielen et al. 2004; Dahle et al. 2005; Manthey et al. 2005). This involves solving for flow using Darcy's law with intrinsic phase average pressure, phase flux, and saturation as the primary variables, and use equivalents of equations (12) and (14) as constitutive relationships. This has the advantage that the intrinsic phase average pressure appears directly as a primary variable in the calculation. However, we have provided a simple example that shows that this approach has serious drawbacks in terms of the constitutive relationships: The capillary pressure includes a term which scales as length squared, and which contains a time derivative of saturation. Further, the relative permeability may exceed unity, and even be discontinuous at the endpoint. In our simple system, this appears as a consequence of the sharp front. Similar phenomena, although to lesser extent, can be expected for other systems on length scales where gradients in saturation are noticeable. Thus we can expect also the relative permeability to exhibit dynamic terms.

Alternatively, the macroscopic system can be solved using the Darcy pressure defined through equations (22) and (24), instead of using the intrinsic phase average pressure as a primary variable. Then the appropriate constitutive relationships are given by equations (27) and (29). This has the significant advantage that the constitutive relationships are simple, without any dynamic terms. This is particularly appealing, since the single tube in consideration here is in a sense the simplest possible porous media. If we cannot obtain simple equations for this porous media, we will face large difficulties when dealing with more complicated systems. The disadvantage with this formulation is obviously that the Darcy pressure is not equal to the intrinsic phase average pressure. In applications where it is important to know the intrinsic phase average pressure precisely, this might be a problem. This disadvantage is somewhat offset by the presence of the formula given in Equation (24). Rewriting this equation as a relationship for the intrinsic phase average

pressure using Equation (27), we see that the intrinsic phase average pressure can be obtained from the Darcy pressure solution through

$$\langle P \rangle^\alpha = [P]_\alpha + (\mathbf{x}^\alpha - \mathbf{x}) \cdot \nabla [P]^\alpha. \quad (30)$$

From a macroscopic viewpoint, the value of this expression is dependent on the availability of a good estimate for the heterogeneity measure  $(\mathbf{x}^\alpha - \mathbf{x})$ . This can be estimated by approximating phase fraction changes as linear (Nordbotten et al. 2006):

$$\mathbf{x}^\alpha - \mathbf{x} \approx C_V \ell^2 \frac{\nabla \epsilon_\alpha}{\epsilon_\alpha} \quad (31)$$

where the constant  $C_V$  is dependent on the shape of the averaging volume. For simple geometric shapes such as spheres ( $C_V = 20$ ) and cubes ( $C_V = 12$ ) it can be obtained analytically, however it is still an open question to what extent this expression can be applied in practical applications.

We finally note that it is important that the choice of constitutive relationships and the interpretation of the macroscopic pressure is consistent throughout the modelling process to get correct results. The usual convention is to neglect dynamic terms, and use a relative permeability function bounded between zero and one. This implies that most simulations today are indeed performed with the Darcy pressure defined in Equation (24). This fact is of utmost importance to acknowledge when using pore-scale models to obtain macroscopic parameters.

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