A TRIPLE-CONTINUUM NUMERICAL MODEL FOR SIMULATING MULTIPHASE FLOW IN VUGGY FRACTURED RESERVOIRS

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ABSTRACT

The existence of vugs (empty holes or cavities) in naturally fractured reservoirs has long been observed and can be attributed significantly to reserves of underground natural resources, such as oil, natural gas, and groundwater. A new multi-continuum conceptual model has been developed for investigating multiphase flow behavior through vuggy fractured reservoirs. The conceptual model, based on geological data and observations of core examples from carbonate formations in China, has been implemented into a three-dimensional, three-phase reservoir simulator using a generalized multi-continuum modeling approach. In this conceptual model, vuggy fractured rock is considered as a triple-continuum medium, consisting of (1) highly permeable fractures, (2) low-permeability rock matrix, and (3) various-sized vugs. The matrix system may contain a large number of small or isolated cavities (of centimeters or millimeters in diameter), whereas vugs are larger cavities, with sizes from centimeters to meters in diameter, indirectly connected to fractures through small fractures or microfractures. Similar to the conventional double-porosity model, the fracture continuum is responsible for the occurrence of global flow, while vuggy and matrix continua, providing large-storage space, are locally connected to each other as well as interacting with globally connecting fractures. In the numerical implementation, a control-volume, integral finite-difference method is used for spatial discretization, and a first-order finite-difference scheme is adapted for temporal discretization of governing flow equations in each continuum. The resulting discrete nonlinear equations are solved fully implicitly by Newton iteration. The numerical scheme has been verified by comparing its results against those of analytical methods for the case of single-phase flow.

1. INTRODUCTION

Characterizing vuggy fractured rock has currently received attention, because many naturally fractured vuggy reservoirs have been found worldwide and can be largely attributed to reserves of underground natural resources, such as oil, natural gas, and groundwater. Significant progress has been made towards the understanding and modeling of flow and transport processes in fractured rock since the 1960s (e.g., Barenblatt et al., 1960; Warren and Root, 1963; Kazemi, 1969; Pruess and Narasimhan, 1985). However, most of those studies have strickly focused on naturally fractured reservoirs, without including cavities. Recently, driven by the need to develop underground natural resources, and by environmental concerns, interest is being generated in investigating vuggy fractured reservoirs (Kossack and Curpine, 2001; Rivas-Gomes et al. 2001; Liu et al., 2003; Hidajat et al. 2004; Camacho-Velazques et al. 2005).

The commonly used mathematical methods for modeling flow through fractured rock include: (1) an explicit discrete-fracture and matrix model (e.g., Snow, 1969), (2) the dual-continuum method, including double- and multi-porosity, dual-permeability, or the more general "multiple interacting continua" (MINC) method (e.g., Warren and Root, 1963; Kazemi, 1969; Pruess and Narasimhan, 1985), and (3) the effective-continuum method (ECM) (e.g., Wu, 2000a). Among these three approaches, the dual-continuum method has been perhaps the most used in application. This is because of its computationally less demanding than the discrete-fracture approach, and it can handle fracture-matrix interactions under multiphase flow, heat transfer, and chemical transport conditions in fractured reservoirs.

Following the traditional double-porosity concept, a number of triple-porosity or triple-continuum models have been proposed (e.g., Closmann, 1975, Wu and Ge, 1983; Abdassah and Ershaghis, 1986; Bai et al., 1993; Wu et al., 2004) for describing flow through fractured rocks. In particular, Liu et al. (2003) and Camacho-Velazquez et al. (2005) present several new triple-continuum models for single-phase flow in a fracture-matrix system that include cavities within the rock matrix (as an additional porous portion of the matrix). In general, these developed models have been focused on handling the heterogeneity of the rock matrix, i.e., subdividing the rock matrix into two or more subdomains with different porous medium properties.

The objectives of this study are (1) to propose a triple-continuum conceptual model to include effects of different-sized vugs and cavities on multiphase flow processes in naturally fractured vuggy rock; (2) to develop a methodology for numerically implementing of the proposed model; and (3) to verity the proposed model. In particular, we discuss the issues and the physical rationale for representing vugs using a numerical approach.

2. TRIPLE-CONTINUUM CONCEPT AND MATHEMATICAL MODEL

As observed in the carbonate formation of the Tahe Oilfield in western China, a typical fractured vuggy reservoir consists of a large and well connected fractured, lower-permeable rock matrix, as well as a large number of cavities or vugs. Those vugs and cavities are irregular in shape and very different in size from millimeter to meters in diameter. Many of the small-sized cavities appear to be isolated from fractures. In this paper, we use “cavities” for those small caves (with sizes of centimeters or millimeters in diameter), while “vugs” represent larger cavities (with sizes from centimeters to meters in diameter). Several conceptual models for vugs are shown in Figures 1, 2, and 3: (1) vugs are indirectly connected to fractures through small fractures or microfractures; (2) vugs are isolated from fractures or separated from fractures by rock matrix; and (3) some of vugs are directly connected to fractures and some are isolated. In reservoirs, there are many more vug varieties and spatial distributions than those shown in Figures 1, 2 and 3, some of which may be approximated by the conceptual models of Figures 1, 2, and 3 or their combinations. In no circumstance, however, is there a need for uniform size distribution patterns for vugs and cavities in this study.

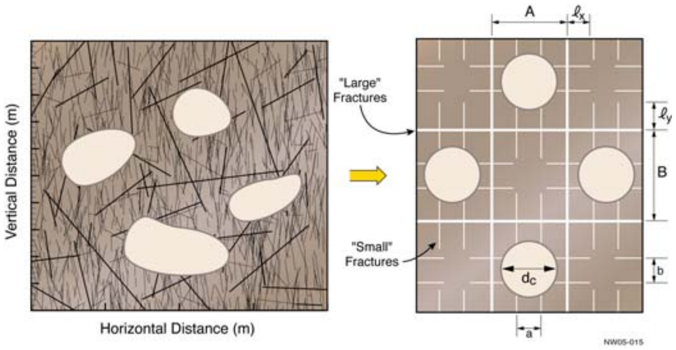

FIGURE 1. Conceptualization#1 of vuggy fractured rock as a triple-continuum system with vugs indirectly connected to fractures through small fractures

Similar to the conventional double-porosity concept (Warren and Root, 1963), large fractures, or the fracture continuum, are conceptualized to be main pathways for global flow, while vuggy and matrix continua, mainly providing storage space as sinks or sources, are locally connected to each other, as well as directly or indirectly interacting with globally connecting fractures. Note that vugs and cavities directly connected with fractures (e.g., Figure 3) are considered part of the fracture continuum. More specifically, we conceptualize the fractured-vug-matrix system as consisting of (1) "large" fractures (or fractures), *globally connected* on the model scale, (2) various-sized vugs, which are *locally connected* to fractures either through "small" fractures or through rock matrix, and (3) rock matrix, which may contain a number of cavities, *locally connected* to large fractures and/or vugs.

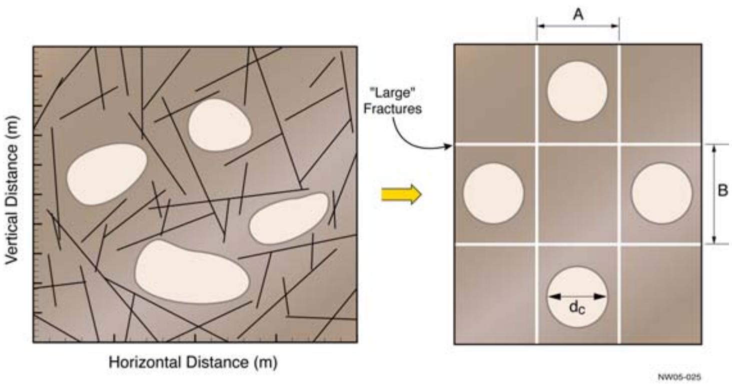

FIGURE 2. Conceptualization#2 of vuggy fractured rock as a triple-continuum system with vugs isolated from or indirectly connected to fractures through rock matrix

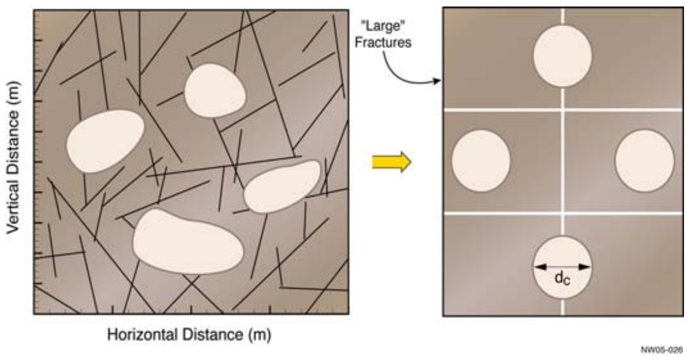

FIGURE 3. Conceptualization#3 of vuggy fractured rock as a triple-continuum system with partial vugs isolated from and certain vugs directly connected to fractures

In principle, the proposed triple-continuum model is a natural extension of the generalized multi-continuum (MINC) approach (e.g., Pruess and Narasimhan, 1985; Wu et al., 2004). In this approach, an "effective" porous medium is used to approximate the fracture, vugs, and rock matrix by considering the three continua to be spatially overlapping. The triple-continuum model assumes that approximate thermodynamic equilibrium exists locally within each of the three continua at all times. Based on this local equilibrium assumption, we can define thermodynamic variables, such as pressures, fluid saturations, and temperatures, for each continuum. Note that the triple-continuum model is not limited to the orthogonal idealization of the fracture system or uniform size or distribution of vugs and cavities, as those illustrated in Figures 1, 2, and 3. Irregular and stochastic distributions of fractures and cavities can be handled numerically, as long as the actual distribution patterns are known.

A multiphase isothermal system in fractured vuggy reservoirs is assumed to be composed of three phases: NAPL (oil), gas, and water. For simplicity, these three components are assumed to be present only in their associated phases with each phase flowing in response to pressure, gravitational, and capillary forces according to Darcy's law. Therefore, three mass-balance equations fully describe the system in an arbitrary flow region of the porous, fractured, vuggy domain:

$$\frac{\partial}{\partial t}(\phi S_\beta \rho_\beta) = -\nabla \bullet (\rho_\beta \mathbf{v}_\beta) + q_\beta \tag{1}$$

where ρ_β is the density of fluid β (β = o, g, or w for NAPL, gas, and water phase, respectively); $\mathbf{v}_f$ is the Darcy (or volumetric) velocity of fluid β; S_β is the saturation of fluid β; ϕ is the effective porosity of formation; t is time; and q_β is the sink/source term of phase β per unit volume of formation, representing mass exchange through injection/production wells or due to fracture-matrix-vug interactions.

Equation (1), governing mass balance for three-phase flow, needs to be supplemented with constitutive equations that express all the secondary variables and parameters as functions of a set of key primary thermodynamic variables. The following relationships will be used to complete the description of multiphase flow through fractured porous media:

$$S_w + S_o + S_g = 1 \tag{2}$$

In addition, capillary pressure and relative permeability relations are also needed for each continuum, which are normally expressed in terms of functions of fluid saturations. The densities of water, NAPL, and gas, as well as the viscosities of fluids, can in general be treated as functions of fluid pressures.

3. NUMERICAL FORMULATION

The governing equations, as discussed in Section 2, for multiphase flow in fractured vuggy reservoirs have been implemented into a general-purpose, three-phase reservoir simulator, the MSFLOW code (Wu, 2000b). As implemented numerically, Equation (1) is discretized in space using an integral finite-difference or control-volume finite-element scheme for a porous-fractured-vuggy medium. Time discretization is carried out with a backward, first-order, finite-difference scheme. The discrete nonlinear equations for water, NAPL, and gas flow at Node i are written as follows:

$$\left\{\left(\phi\, S_\beta \rho_\beta\right)_i^{n+1} - \left(\phi\, S_\beta \rho_\beta\right)_i^{n}\right\}\frac{V_i}{\Delta t} = \sum_{j\in\eta_i} F_{\beta,ij}^{n+1} + Q_{\beta i}^{n+1} \tag{3}$$

where superscript n denotes the previous time level; n+1 is the current time level; V_i is the volume of element i (porous or fractured block); Δt is time step size; η_i contains the set of neighboring elements (j) (porous, vuggy, or fractured) to which element i is directly connected; $F_{\beta,ij}$ is the mass flow term for phase β between elements i and j; and $Q_{\beta i}$ is the mass sink/source term at element i, of phase β.

The "flow" term ($F_{\beta,ij}$) in discrete Equation (3) for multiphase flow between and among the triple-continuum media, along the connection (i, j), is given by

$$F_{\beta,ij} = \lambda_{\beta,ij+1/2}\gamma_{ij}\left[\psi_{\beta j} - \psi_{\beta i}\right] \tag{4}$$

where $\lambda_{\beta,ij+1/2}$ is the mobility term to phase β, defined as

$$\lambda_{\beta,ij+1/2} = \left(\frac{\rho_\beta k_{r\beta}}{\mu_\beta}\right)_{ij+1/2} \tag{5}$$

Here subscript ij+1/2 denotes a proper averaging or weighting of properties at the interface between two elements i and j; and $k_{r\beta}$ is the relative permeability to phase β. In Equation (4), γ_{ij} is transmissivity and is defined, if the integral finite-difference scheme (Pruess et al., 1999) , as

$$\gamma_{ij} = \frac{A_{ij} k_{ij+1/2}}{d_i + d_j} \tag{6}$$

where A_{ij} is the common interface area between connected blocks or nodes i and j; d_i is the distance from the center of block i to the interface between blocks i and j; and $k_{ij+1/2}$ is an averaged (such as harmonic weighted) absolute permeability along the connection between elements i and j. The flow potential term in Equation (4) is defined as

$$\psi_{\beta i} = P_{\beta i} - \rho_{\beta,ij+1/2}\, g\, D_i \tag{7}$$

where D_i is the depth to the center of block i from a reference datum. The mass sink/source term at element i, $Q_{\beta i}$ for phase β, is defined as

$$Q_{\beta i} = q_{\beta i} V_i \tag{8}$$

Note that Equation (3) has the same form regardless of the dimensionality of the model domain, i.e., it applies to one-, two-, or three-dimensional analyses of multiphase flow through vuggy fractured porous media. In our numerical model, Equation (3) is written in a residual form and is solved using Newton/Raphson iteration fully implicitly.

4. HANDLING FRACTURES AND VUGS

The technique used in this work for handling multiphase flow through vuggy fractured rock follows the dual- or multi-continuum methodology (Warren and Root, 1963; Pruess and Narasimhan, 1985). With this dual-continuum concept, Equations (1) and (3) can be used to describe multiphase flow along fractures, inside matrix blocks, as well as fracture-matrix-vug interaction. However, special attention needs to be paid to treating inter-porosity flow in the fracture-matrix-vug triple continua. Flow terms between fracture-matrix, fracture-vug, and vug-matrix connections are all evaluated using Equation (4). However, the transmissivity of (6) will be evaluated differently for different types of inter-porosity flow. For fracture-matrix flow, γ_{ij}, is given by

$$\gamma_{FM} = \frac{A_{FM} k_M}{l_{FM}}, \tag{9}$$

where A_{FM} is the total interfacial area between fractures (F) and the matrix (M) elements; k_M is the matrix absolute permeability; and l_{FM} is the characteristic distance for flow crossing fracture-matrix interfaces. For fracture-vug flow, γ_{ij} is defined as

$$\gamma_{FV} = \frac{A_{FV} k_V}{l_{FV}} \tag{10}$$

where A_{FV} is the total interfacial area between the fracture (F) and vugs (V) elements; l_{VM} is a characteristic distance for flow crossing vug-matrix interfaces; and k_V is the absolute vuggy permeability, which is the actual permeability of small fractures that control flow between vugs and fractures (Figure 1). Note that for the case in which vugs are isolated from fractures, as shown in Figures 2 and 3, no fracture-vug flow terms need to be calculated.

For vug-matrix flow, γ_{ij} is evaluated as

$$\gamma_{VM} = \frac{A_{VM} k_M}{l_{VM}} \tag{11}$$

where A_{VM} is the total interfacial area between the vug (V) and matrix (M) elements; and l_{VM} is a characteristic distance for flow crossing vug-matrix interfaces.

Note that Table 1 summarizes several simple models for estimating characteristic distances in calculating inter-porosity flow within fractures, vugs, and the matrix, in which case we have regular one-, two-, or three-dimensional large fracture networks, each with uniformly distributed small fractures connecting vugs or isolated vugs from fractures (Figures 1, 2, and 3). The models in Table 1 rely on the quasi-steady-state flow assumption of Warren and Root (1963) to derive characteristic distances for flow between fracture-matrix and (through small fractures) fracture-vug connections. Another condition for using the formulation in Table 1 is that fractures, vug, and matrix are all represented by only one gridblock. In addition, the flow distance between large fractures (F) and vugs (V), when connected through small fractures, is taken to be half the characteristic length of the small fractures within a matrix block (Figure 1). Furthermore, the interface areas between vugs and the matrix should include the contribution of small fractures for the case of Figure 1. Interface areas between fractures and the matrix, and between fractures and vugs through connecting small fractures, should be treated using the geometry of the large fractures alone. This treatment implicitly defines the permeabilities of the fractures in a continuum sense, such that bulk connection areas are needed to calculate Darcy flow between the two fracture continua.

TABLE 1. Characteristic distances* for evaluating flow terms between fractures, vugs, and matrix systems

Fracture Sets	Dimensions of Matrix Blocks (m)	Characteristic F-M Distances (m)	Characteristic F-V Distances (m)	Characteristic V-M Distances [1] (m)	Characteristic V-M Distances [2] (m)
1-D	A	$l_{FM} = A/6$	$l_{FV} = l_x$	$l_{VM} = a/6$	$l_{VM} = (A - d_c)/2$
2-D	A, B	$l_{FM} = AB / 4(A+B)$	$l_{FV} = \frac{l_x + l_y}{2}$	$l_{VM} = ab / 4(a+b)$	$l_{VM} = \frac{A+B-2d_c}{4}$
3-D	A, B, C	$l_{FM} = 3ABC/10 /(AB+BC+CA)$	$l_{Ff} = \frac{l_x + l_y + l_z}{3}$	$l_{VM} = 3abc/10 /(ab+bc+ca)$	$l_{Ff} = \frac{A+B+C\text{-}3d_c}{6}$

* Note in Table 1, A, B, and C are dimensions of matrix blocks along x, y, and z directions, respectively.
[1] Characteristic V-M distances are estimated for the case (Figure 1), i.e., vuggy-matrix connections are dominated by small fractures, where dimensions a, b, and c are fracture-spacings of small fractures along x, y, and z directions, respectively.
[2] Characteristic V-M distances are used for the case (Figures 2 and 3), i.e., vugs are isolated from fractures.

The MINC concept (Pruess, 1983) is used to generate a triple-continuum grid, which is a key step in modeling flow through fractured-vuggy rock. We start with a primary or single-porous medium mesh that uses bulk volume of formation and layering data. Then, we use geometric information for the corresponding fractures and vugs within each formation subdomain or each finite-difference gridblock of the primary mesh. Fractures are lumped together into the the fracture continuum, while vugs with or without small fractures are lumped together into the vuggy continuum. The rest is treated as amatrix continuum. Connection distances and interface areas are then calculated accordingly, e.g., using the relations listed in Table 1 and the geometric data of fractures. Once a proper mesh for a triple-continuum system is generated, fracture, vuggy, and matrix blocks are specified, separately, to represent fracture or matrix continua.

5. COMPARISON WITH ANALYTICAL SOLUTION

We examined the numerical model using an analytical solution (Lui et al, 2003; Wu et al. 2004). The verification problem concerns typical transient flow towards a well that fully penetrates a radially infinite, horizontal, and uniformly vuggy fractured reservoir. Numerically, a radial reservoir (r_e = 10,000 m) of 20 m thick is represented by a 1-D (primary) grid of 2,100 intervals. A triple-continuum mesh is then generated using a 1-D vuggy-fracture-matrix conceptual model, consisting of a horizontal large-fracture plate network with a uniform disk-shaped matrix block. Uniform spherical vugs are contained inside the matrix and connected to fractures through small fractures. Fracture, vugs and matrix parameters are given in Table 2.

TABLE 2. Parameters used in the single-phase flow problem in the triple-continuum, vuggy fractured reservoir

Parameter	Value	Unit
Matrix porosity	$\phi_M = 0.263$	
Fracture porosity	$\phi_F = 0.001$	
Vuggy porosity	$\phi_V = 0.01$	
Fracture spacing	$A = 5$	m
Small-fracture spacing	$a = 1.6$	m
F characteristic length	$l_x = 3.472$	m
F-M/F-V areas per unit volume rock	$A_{FM} = A_{FV} = 0.61$	m^2/m^3
Reference water density	$\rho_i = 1{,}000$	kg/m^3
Water phase viscosity	$\mu = 1\times10^{-3}$	Pa•s
Matrix permeability	$k_M = 1.572\times10^{-16}$	m^2
Fracture permeability	$k_F = 1.383\times10^{-13}$	m^2
Small-fracture or vug permeability	$k_V = 1.383\times10^{-14}$	m^2
Water Production Rate	$q = 100$	m^3/d
Total compressibility of three media	$C_F=C_M=C_V= 1.0\times10^{-9}$	1/Pa
Well radius	$r_w = 0.1$	m
Formation thickness	$h = 20$	m

Figure 4 compares numerical-modeling results with the analytical solution for a single-phase transient flow case (in terms of dimensionless variables). Excellent agreement exists between the two solutions, which provides verification of the numerical formation and its implementation. Note that there are very small differences at very early times (t_D < 10 or 0.2 seconds) in the two solutions in Figure 4, which may occur because the analytical solution, which is long-time asymptotic and similar to the Warren-Root solution, may not be valid for t_D < 100.

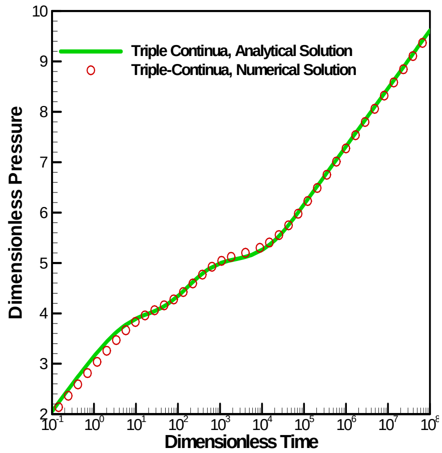

FIGURE 4. Comparison between analytical and numerical solutions for single-phase transient flow through vuggy fractured formation.

6. SUMMARY AND CONCLUDING REMARKS

We have developed a physically based conceptual model for modeling multiphase flow through vuggy fractured rock. In this conceptual model, vuggy fractured rock is represented by a triple-continuum medium of fractures, rock matrix, and vugs. This is a natural extension of the classic double-porosity model, with the fracture continuum responsible for conducting global flow, while vuggy and matrix continua, providing storage space, are locally connected as well as interacting with globally connecting fractures.

The proposed conceptual model has been implemented into a general numerical reservoir simulator using a control-volume, finite-difference approach, which can be used to simulate single-phase as well as multiple-phase flow in 1-D, 2-D and 3-D reservoirs. In addition, a verification example is provide for the numerical scheme by comparing numerical results against an analytical solution for single-phase flow. The model's application to actual vuggy-fractured petroleum reservoirs is under way.

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